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ARCHIMEDISM, NOT PLATONISM

On a malleable ideology of Renaissance mathematicians (1400 to 1600), and on its role in the formation of seventeenth-century philosophies of science

By JENS HØYRUP

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preprint

DEDICATED TO MARSHALL CLAGETT

for reasons that will be obvious to everybody familiar with the Medieval Archimedes

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I. The question of Platonism

We all know the following set of stereotypes:

- Scholastic natural philosophy was Aristotelian and therefore non-mathematical.
- The Renaissance and the Humanist movement rediscovered Plato, and were fundamentally and strongly Platonic.
- Plato held a mathematical view of the world.
- The Renaissance produced a fresh start in mathematics and in mathematicized natural philosophy.

It is easy to array arguments in favour of these stereotypes, and even if they represent violent simplifications (especially in their undifferentiated view of »Scholasticism« and »Renaissance«) they are far from being totally mistaken. It is therefore attractive to combine them into a stereotype conclusion, *viz.* that

- Renaissance mathematics and mathematicized science were expressions and consequences of Renaissance Platonism.

It will be my first aim in the following to demonstrate that this conclusion is far less true than the partially true premisses from which it is drawn (no unusual situation within the polyvalent logic so characteristic of historiography turning explanatory). My second aim shall be to demonstrate that another intellectual giant of Antiquity can be argued with much better reason to have served as an example and as a navigation mark to Renaissance mathematicians,

and that *Archimedism* provided them with an ideology almost as Protean as the Platonism of miscellaneous philosophical currents yet serviceable as inspiration and guide-post and for professional identity formation.

First to the question of Platonism. It would be easy to point to large amounts of definitely non-Platonic mathematical activity made during the Renaissance era (say, 1400 to 1600). The whole quadrivial tradition as it lingered on at many universities until well into the sixteenth century could be used for that purpose; most German Rechenmeister are also pretty unphilosophical, as is much of their Italian model, the tradition of the Trattati d'abbaco. The existence of these traditions, however, is a trivial and rather uninteresting point. The real question is whether the renewal in Renaissance mathematics—renascent mathematics—was Platonic.

The delimitation of the *new* tendencies is in itself a problem, which I shall answer—or, it may be thought, avoid—below by discussing a selection of outstanding examples. But at first any claim about »Platonism« requires that we clarify the meaning of this term.

The most obvious sense in which somebody could be considered a Platonist is if he adheres to (some interpretation of) the complete philosophical system. But this sense has little meaning in the present

¹ The starting point for my work on the present paper was a larger investigation of the influence of philosophical currents and quasi-philosophical attitudes on the changing styles, ideals and norms for mathematical activity in Latin and Renaissance Europe from the early twelfth to the outgoing sixteenth century (Høyrup 1987 or 1987a). Because of its narrower focus, the present investigation draws in part on sources not mentioned in these earlier publications; many general theses on the character and subdivisions of Renaissance mathematics dealt with there, on the other hand, are taken for granted in the present essay.

All translations from the sources in the following are mine, unless the contrary is stated explicitly.

Richard Lorch looked through a preliminary version of the paper for linguistic oddities, for which it is a great pleasure to express my gratitude. Since I provided the initial conditions for the weeding; since I did not follow his advice in all cases; and since I made changes in the text following upon his corrections, nobody but I am of course responsible for the errors which remain.

connection, since the above-quoted stereotype »Plato held a mathematical view of the world« is only a half-truth. The *Timaeus*, admittedly, proposes a cosmology founded upon geometrical atomism. In other more central works, however, mathematics is only considered a *first*, approximate model for true, dialectical knowledge, a means for »the awakening of thought« (*Republic* 523A). This Platonism does not turn up in Renaissance mathematical writings, although traces of it can be found in certain *philosophical* writings, e.g. in Ficino's *Liber de vita*².

The general Neo-Platonism of Ficino's circle is no more relevant. Its interest in Hermes Trismegistos and in the *Prisca theologia* is shared by a few mathematical writers (among them Lefèvre d'Étaples and Foix de Candale, cf. below), but it remained peripheral, being usually combined with kabala and numerology and not with genuine mathematics.

Mathematicians could also be considered Platonists if they were or believed themselves to be inspired by a particular part of the Platonic corpus, either to do mathematics at all or in the choice of a particular approach to the subject. The latter possibility can be excluded immediately: No part of the corpus is precise enough in its description of mathematics to convey a mathematically specific message to a generation which had understood its Euclid. The former possibility is open in principle but closed *de facto*. Admittedly, many writers refer to Platonic passages in order to legitimate the pursuit of mathematics. But Plato is never alone in the works which I have looked at, and never of outstanding importance.

² »It would be useful, too, to look at a sphere with its motions, as Archimedes once did, and which recently a certain Florentine by the name of Lorenzo has made. Not just to look at it, but to reflect on it in the soul« (trans. Boer 1980: 153)—one of the two passages where a mathematician different from Pythagoras and Ptolemy is mentioned (the other regard Archytas' automata).

Neither in the former (strong) nor in the latter (diluted) sense is it thus possible to verify the claim that Renaissance mathematics was Platonist. Why then is the claim so persistent?

The real intention of the claim seems to be a contrast. Thirteenth to fourteenth century mathematics may not always have been Aristotelian in the strong sense³; but in the diluted sense it was—vide, e.g., the recurrent lip-service offered to the four Aristotelian Causes in the most unlikely connections. To claim that Renaissance mathematics was Platonist should read as a claim that mathematics was no longer (diluted) Aristotelian.

The reason to identify non-Aristotelianism with Platonism is not exclusively endorsement of the dichotomizing topos knowing but these two directions in philosophy (even though this fallacy certainly plays a role). To see what more there is to the claim for Platonist Renaissance mathematics we may look at the material presented by A. C. Crombie in an article (1977) on sixteenth century "Mathematics and Platonism ...". Renazzi's early nineteenth century history of the University of Rome is quoted for the claim that a number of Roman sixteenth century mathematicians were Platonists; most vivid is the statement that Giambattista Raimondi (c. 1536-1614) »led the way through his lectures in toppling Aristotle from the philosophical throne and replacing him by Plato« (trans. Crombie 1977: 63). In the corresponding note, on the other hand, a work by Girolamo Lunadoro from 1635 is quoted for the information that Raimondi had »beautiful thoughts about the doctrine of Plato, and that of Aristotle, as he was very well versed in both of these authors «4. Quotations from the writings of Clavius (1537-1612) and Possevino (1533/34-1611) carry the same message. In his discussion of "The Way in

³ As demonstrated in my (1987: 17-31), thirteenth century mathematics was mostly *not*, even though certain philosophical writings *about* mathematics were.

^{* »...} belli pensiero circa la doctrina di Platone, e di Aristotele, per essere versatissimo, in ambi due questi autori«—Crombie 1977: 78 n.6.

Which the Mathematical Disciplines Could be Promoted in the Schools of the [Jesuit] Society" Clavius refers to »the infinite examples in Aristotle, Plato and their more celebrated commentators, which can by no means be understood without a moderate understanding of the mathematical sciences«, whence »teachers in philosophy should be skilled in mathematical disciplines, at least moderately« (trans. Crombie 1977: 65f); in the Jesuit Ratio studiorum from 1586, strongly influenced by Clavius, mathematics is told to assist the Analytics by providing »examples of solid demonstrations«, and to be useful for metaphysics by telling »the number of spheres and intelligences« (trans. Crombie 1977: 66)-and for poets, for historians, for politicians, for physics, for theologians, for law and ecclesiastical custom, and for the state, all for various more or less plausible reasons; Possevino, finally, in the chapter on mathematics (which Clavius had co-authored) of his encyclopedic Bibliotheca selecta from 1587-91, cites Plato's Timaeus and Aristotle's Physics as »very great proof of how much light mathematics itself sheds on philosophy« (trans. Crombie 1977: 70).

»Platonism« is thus not unspecific »non-Aristotelianism«; it reduces to that contrast to strict (Averroist and similar) Aristotelianism which regarded Plato and Aristotle as nothing but reconcilable exponents of Ancient Wisdom. In this sense, of course, many Renaissance mathematicians were just as much Platonists as the majority of Renaissance thinkers in general. But this is a »Platonism« where the specific character of Plato's own philosophy had largely disappeared; its chief argument for the philosophical importance of mathematics, moreover, is the necessity of fundamental mathematical knowledge for anybody who wants to enter the world of philosophical authors (no wonder that the favourite quotation from Plato on mathematics is the legendary »Let nobody unskilled in geometry enter«). In practice, mathematics is thus a first philosophy

common to Plato, Aristotle, Ciceronian Stoicism, etc., and its necessity not to be derived from a particular system⁵.

It seems to be a legitimate conclusion that the »Platonism« of Renaissance mathematicians is nothing but a red herring diverting the hounds from the track. It has little to do with any serious philosophical Platonism (not even the specific Platonisms of the Renaissance); it was only one ingredient in a more general reverence for Ancient wisdom *in toto*; and it only influenced the way mathematics was done quite superficially, namely by legitimating the mathematical enterprise as a whole.

But where then should we look for the fox?

II. Renascent mathematics

Not all mathematical activity of the Renaissance era was involved in renewal and in break with the Medieval past. As already mentioned, much fifteenth century university mathematics and much of what went on the vicinity of the abacus school would only blur the picture if we look for the characteristics of »renascent mathematics«. Only by concentrating on the latter current (in so far as it is permissible to speak about *one* current) will we be able to distinguish its particular traits.

⁵ This role as a philosophical *sine qua non* is different from and much more ambitious than that of being a preliminary preparation of the mind, i.e., from the role ascribed to mathematics by Plato.

Renascent mathematics is mathematics conscious of its own rejuvenating role, and thus mathematics somehow connected with the Humanist movement. An appropriate first example will therefore be Leone Battista Alberti (1404-1472), who was not only a mathematician but also a major Humanist.

One of his mathematical works is the Ludi rerum mathematicarum (ed. Grayson 1973: 131-173), a recreational opuscule mainly on practical geometry and dedicated »all'umanitá e facilità« of the princely recipient. It thus unites civic utility with noble leisure; or, to be more precise, presents dilettante interest in a subject of public utility as a noble way to fill out leisure—in full harmony with the tendency of fiftenth century Italian Humanism.

Aristotle is not mentioned in the work, and neither is Plato or Euclid⁶. Only Archimedes, »uomo suttilissimo«, turns up (p. 172), viz. because of his exposure of the fraud committed by Hieron's goldsmith.

In the dedicatory letter of the Italian version of the *De pictura*, Alberti tells more about his conception of the mathematical sciences: They are among those »elevated and divine arts and sciences« which had flourished in Antiquity but now were »missing and almost

⁶ None of them, in fact, are mentioned anywhere in the mathematical works—except for the claim in *De pictura II*, 27 (ed. Grayson 1973: 48) that Plato, together with Socrates and others, »furono in pittura conosciuti«. The absence of Euclid is all the more remarkable since one of Alberti's mathematical works is the *Elementa picturae*, of obvious Euclidean inspiration.

The De re aedificatoria (ed., trans. Theuer 1912) contains 19 references to Aristotle and 24 to Plato. In so far as these are not purely anecdotic, however, they refer to Plato the city-planner and the social planner, and to Aristotle the natural historian or the city-planner. None of them stand out as authorities, both are authors with whom one may agree or—if needed—disagree.

Archimedes is mentioned only a few times in the latter work: Three times in connection with the movement of very large weights, and once as a representative of that sophistication in the treatment of angles and lines to which the architect should *not* aspire (Theuer 1912: 519).

completely lost«: painting, sculpture, architecture, music, geometry, rhetorics, augury and similar noble and wonderful undertakings⁷.

No doubt, then, that for Alberti the resurrection of mathematics was in itself a legitimate resurrection of Ancient splendour. He did not need, and made no appeal to, any philosophical legitimation for his efforts.

Another architect-mathematician of somewhat later date is Luca Pacioli (c. 1445-1517). He was more loquacious than most mathematicians, and his *De divina proportione* (written 1496-97, published 1509; ed., tr. Winterberg 1896) contains an introduction to his views of mathematics extending over several chapters. The relation of the subject to philosophy is made clear by the claim that his work is necessary to »everybody wanting to study philosophy, perspective, painting, sculpture, architecture, music, and other most pleasant, subtle and admirable doctrines«, and no less by the statement that

the mathematical sciences of which I speak are the fundament for and the ladder by which one arrives at knowledge of any other science, because they possess the first degree of certitude, as the philosopher says when claiming that "the mathematical sciences are in the first degree of certitude, and the natural sciences follow next to them". As stated, the mathematical sciences and disciplines are in the first degree of certitude, and all the natural sciences follow from them. And without knowing the mathematical sciences it is impossible to understand any other well. In Solomon's Wisdom it is also written that "everything consists in number, weight and measure", that is, everything which is found in the inferior or the superior universe is by this necessity submitted to number, weight and measure. And Aurelius Augustine says in De civitate Dei that the supreme artisan should be supremely praised because "in them he made

⁷ »Io solea maravigliarmi insieme e dolermi che tante ottime e divine arti e scienze, quali per loro opere e per le istorie veggiamo copiose erano in que' vertuosissimi passati antiqui, ora così siano mancate e quasi in tutto perdute: pittori, scultori, architetti, musici, ieometri, rhetorici, auguri e simili nobilissimi e maravigliosi inteletti oggi si truovano rarissimi e poco da lodarli«—ed. Grayson 1973: 7.

exist that which was not«8.

For Luca, then, mathematics is as much the real first philosophy as it was to be for Clavius and his associates; to drive home the point he even enlists Aristotle, twisting his "being secondary to mathematics in exactness" to mean "follow by logical derivations from mathematics". But mathematics is more than that, and in the following passage Luca describes the utility of mathematics in warfare—no doubt more interesting to Ludovico Sforza of Milano, for whom the work is written. Here Archimedes is brought into the argument, "el nobile ingegnoso geometra e dignissimo architetto" and "gran geometra". He is the man who provides the connection between mathematics and its civic uses through his legendary defence of Syracuse, which also corresponds to

the daily experience of your Ducal Highness [...] that the defence of large and small republics, called by another name the military art, is impossible unless the knowledge of Geometry, Arithmetic and Proportion can be applied eminently and with honour and utility.9

Alberti and Luca represent the level of distinguished but not eminent fifteenth century Humanist mathematics. A representative of

⁸ »Conciosia che dicte mathematici sienno fondamento e scala de peruenire a la notitia de ciascun altra scientia per esser loro nel primo grado de la certeza affermondolo el philosopho così dicendo. Mathematice enim scientie sunt in primo gradu certitudinis et naturales sequuntur eas. Sonno commo edicto le scientie e mathematici discipline nel primo grado de la certezza e loro sequitano tutte le naturali. E senza lor notitia fia impossibile alcunaltra bene intendere e nella sapientia ancora e scripto. quod omnia consistunt in numero pondere et mensura cioe che tutto cio che per lo vniuerso inferiore e superiore si squaterna quello de necessita al numero peso e mensura fia soctoposto. E in queste tre cose laurelio Augustino inde ciuitate dei dici el summo opefici summamente esser laudato perche in quelle fecit stare ea que non erant« (ed. Winterberg 1896: 36). The claim of the necessity for »ciascun studioso di Philosophia, Perspectiua, Pictura, Sculptura, Architectura, Musica e altre Mathematiche suauissima sottile e admirabile doctrina« is taken from the title page, p. 18.

^{* »...} per quotidiana experientia a Vostra Ducal celsitudine non e ascosto, [...] che la deffensione de le grandi e piccole republiche per altro nome arte militare apellata non e possibili senza la notitia de Geometria Arithmetica e Proportione egregiamente poterse con honore e vtile exercitare« (ed. Winterberg 1896: 37).

the truly eminent level (and probably the most eminent of all) is Regiomontanus (1436-1476), whose opinions on mathematics are perhaps expounded most clearly in a lecture »explaining briefly the mathematical sciences and their utility«, held in Padua in 1463/1464 (printed by Schöner in 1537, facsimile in Schmeidler 1972). Following upon a general introduction comes a history of the subject, in which it is said, after the presentation of Euclid and his Medieval translators, that this »father of all geometers« was

followed by Archimedes citizen of Syracuse, and by Apollonios Pergaeos, customarily called the Divine because of the height of his genius, of whom it is not easy to say whether one is to be preferred to the other. While namely Apollonios described the elements of conics in eight books, which have never been put into Latin, the first rank appears to be given to Archimedes the Sicilian by the variety of publications, which under Pope Nicholas V were rendered in Latin by a certain Jacobus of Cremona; he composed two books on the Sphere and the Cylinder, two on Conoids and Sphaeroids, and as many on Equilibrium; he also wrote about Spiral Lines, where he undertakes to designate a straight line equal to the circumference of the circle, thus permitting the squaring of the circle, which on their part several very ancient philosophers had sought for but nobody had found out until the time of Aristotle, and for the glory of which several most distinguished men of our own age stand ready [apparently a hint to Nicholas of Cusa]. From Archimedes we have, moreover, the Measurement of the Circle, the Quadrature of the Parabola, and the Sandreckoner. Some would claim that he has written a work on Mechanics, where he collects select devices for various uses, on Weights, on Aqueducts, and several others which until now it has not been possible to see.10

¹⁰ »Haec de patre omnium Geometrarum Euclide, cui succedunt Archimedes Siracusanus ciuis, et Apollonius Pergaeus ob ingenij altitudinem diuinus uocari solitus, quorum uter alteri praeferendus sit, non facile dixero. Nam etsi Apollonius elementa conica in octo libris, quos nondum uidit latinitas, subtilissime conscripserit, Archimedi tamen Siculo uarietas rerum editarum principatum contulisse uidetur, quem sub Nicolao quinto Pontifice Iacobus quidam Cremonensis Latinum ex Graeco reddidit, duos de sphaera, et chylindro libros composuit, de Conoidalibus et Sphaeroidalibus duos, totidem de aequeponderantibus, scripsit item de lineis spiralibus, ubi circumferentiae circuli

Later in the lecture Regiomontanus comes to the relations of mathematics to philosophy. Once again mathematics comes in as practical first philosophy while being itself above the judgment of philosophies. As with Luca, mathematics possesses »the first degree of certitude«, and nobody will understand Aristotle who does not know the liberal arts of the quadrivium, neither *De caelo* or *Meteora*, nor *Physica* or *Metaphysica*. Philosophy is split nowadays into warring schools, and victory comes from mastery of sophisms. On *mathematics*, on the other hand, nobody but the insane would dare say something similar, and

neither time nor human customs can detract from it validity. Euclid's theorems have the same certitude today as a thousand years ago. Archimedes' discoveries will inspire no less admiration in the men to come after a thousand generations than pleasure in us when we read about them.¹¹

It is therefore a triumph of the present day that so many theologians and other eminent men, from Bessarion and Nicholas de Cusa to Alberti, are interested in the subject.

In spite of all differences of level and orientation, Alberti, Luca Pacioli and Regiomontanus thus agree on several important points. Firstly, mathematics does not derive its justification from any particular philosophy; but part of its justification comes from its

aequalem rectam designare conatur, quatenus circulum quadrare liceat, quod quidem plerisque uetustissimis philosophis quaesitum est, ad tempora usque Aristotelis autem à nemine compertum, cuius rei gloriam nonnulli nostra tempestate uiri clarissimi praestolantur. Archimede insuper mesurationem circuli accepimus, quadraturam, parabolae et arenae numerum. Sunt qui scripsisse eum asserant Moechanicam, ubi electissima ad uarios usus colligit ingenia, de ponderibus, de aqueductibus, et caeteris quae usquehac uidere non licuit« (ed. Schmeidler 1972:45; not fully literal translation).

11 »Quod de nostris disciplinis nemo nisi insanus praedicare ausit, quandoquidem neque aetas neque hominum mores sibi quicquam detrahere possunt: Theoremata Euclidis eandem hodie quam ante mille annos habent certitudinem. Inuenta Archimedis post mille secula uenturis hominibus non minorem inducent admirationem, quam legentibus nobis iucunditatem« (ed. Schmeidler 1972: 51).

utility for philosophy in general. Apart from that, its civic utility and its root in Antiquity makes it a sublime subject.

One figure is mentioned by all of them as a token: Archimedes. To Alberti and Luca he is first of all a magnificent engineer and architect, to be extolled for his service to his king and for his ingenious mathematical machines. Regiomontanus, the most illustrious mathematician of the century, praises him as a theoretical mathematician and dismisses his engineering feats as undocumented in extant works (and, less directly expressed, as less interesting than his mathematical achievements)¹².

As amply demonstrated by the first volumes of Marshall Clagett's Archimedes in the Middle Ages, Regiomontanus and his contemporaries were not the first Western mathematicians after the Classical era to acknowledge the importance and eminence of Archimedes' mathematics. But his scholastic admirers were not interested in the historical person, and did not endow him with the status of a symbol. That idea seems to have come from the biographical interest of the Roman epoch and its revival in early Humanism. Already in Antiquity the subtlety of Archimedes had become proverbial (see Simms 1989). Basing himself on Cicero, Livy, Firmicus Maternus and other Latin authors, Petrarch (1304-1374) had written several biographical notices on Archimedes (quoted in Clagett 1978: 1336-1340). They describe how Archimedes' absorption in geometrical

¹² Even Regiomontanus, it is true, is more interested in his results than in his ingenious and sophisticated methods. As expressed by Clagett (1978: 383) it »is regrettable that Regiomontanus did not live long enough to exploit his very promising beginning in the study of Archimedes. If he had lived, possessing as he did the techniques and abilities in language and mathematics, he might have been able to anticipate the mastery of the Archimedean corpus first achieved in the sixteenth century by Maurolico and Commandino. As it was, by the time of his death, Regiomontanus had not developed any interest in the higher geometrical problems susceptible to the method of exhaustion«—and given his primary interest in astronomy and the mathematics of astronomy it is perhaps to be doubted whether a longer life would have changed the focus of his mathematical interest.

thought caused his death; the wonderful machines he made for the defence of his city; and his mechanical model of the heavenly system. Further, they speak about his interest in the heavens and his preeminence as an astrologer (claimed by Firmicus Maternus); and do not mention a single mathematical work, not even that Measurement of the Circle which had gone into the biographic notices written by Vincent of Beauvais and other scholastic authors.

The reason that Humanist mathematicians might adopt Archimedes as the supreme representative of their art was thus that Humanist culture was from its beginnings aware of his importance though not of its foundation. An initial reason that neither Euclid nor Apollonios could be adopted in a similar way may be the lack of adequate biographical substance and thus their absence from the Humanists' picture of Antiquity (later on, of course, the respective levels of Euclid and Archimedes would ensure that Euclid could not take over the role as the paragon of mathematics).

The general status of Archimedes even outside the mathematical environment is illustrated by the pet name given by the citizens of early fifteenth century Siena to their foremost engineer Mariano Taccola (1382 to c. 1453). If he was called "the Archimedes of Siena" (Prager and Scaglia 1972: 17 and passim) it was certainly not because of mathematical proficiency—he was no mathematician and never mentions neither Archimedes nor Euclid in his writings (ibid., 156). But he was a skillful designer of military and other engines and a fine artist, which was sufficient to justify the illustrious title.

In spite of the preponderant interpretation of Archimedes as a supreme engineer the veneration of his name also affected the development of fifteenth-century mathematics proper. If Archimedes was the great geometer of Antiquity and his works were still extant, it would, firstly, be important to possess a Latin translation

unpolluted by the stylistic errors of Scholasticism³. Secondly, it would be important for those who took up work in his field (or who wanted an all-encompassing familiarity with Ancient culture) to know him. Though mathematically incompetent in its beginning, Humanist Archimedism was thus a spur to translations and manuscript collection, concerned not only with Archimedes himself but (by way of the general justification of mathematics which it implied) also of other Ancient mathematicians. Finally, it was an incentive for working mathematicians to take up as much of Archimedes as they were able to-who knows whether Regiomontanus the astronomer would have set himself the task14 of publishing Archimedes and Apollonios if he had not been influenced by Bessarion's circle? Helmuth Größing (1980:74f), at least, suggests that this encounter changed both his mathematical and his stylistic ideals; indeed, the contrast between his actual mathematical interests and his enthusiasm for Archimedes suggests that the latter was imposed by more general ideals.

¹³ »... even if [Petrarch's] Archimedes is a long way from the rigorous and certain geometer extolled by later mathematicians, nevertheless the humanistic Archimedes is one foundation of the Archimedean tradition of the Renaissance«, as formulated by P. L. Rose (1975: 9).

¹⁴ Never fulfilled, one will remember, because of Regiomontanus' early and sudden death. But his publishing plans can be consulted in a circular printed in facsimile by Schmeidler (1972: 533).

III. The mature Italian Renaissance

I shall not go into the details of these processes of translation, manuscript collection etc., which have been expounded by Rose (1975: 26-56, passim; 1973). Instead, I shall look at the way Archimedes is seen by some of the mathematicians and the mathematical dilettanti of the sixteenth century. First, however, I shall point to a change undergone by the topos "the Archimedes of ..." during the Renaissance period and its immediate sequel. In the earlier fifteenth century, we remember, it had been used of (and, in pride, by) a skillful but mathematically rather uninformed engineer. In the later sixteenth century it was used by Jean Bodin of François Foix de Candale because of his contributions to pure geometry. Galileo, finally, uses it twice in the Discorsi¹⁶ of Luca Valerio, author of a work on centres of gravity based on the Archimedean finitist method of limits¹⁷.

The changing use of the topos illustrates the gradual shift undergone by the Archimedist idea during the late Renaissance, which will be more clearly visible if we take a closer look at the attitude to Archimedes as expressed by some significant mathematical authors. For this purpose, Cardano will provide us with an adequate starting point. His was a profuse mind, and we

^{15 »}Le grand Archimède de nostre age«—quoted from Westman 1977: 42.

¹⁶ »Sig. Luca Valerio, nuovo Archimede dell'età nostra«, and »Luca Valerio, altro Archimede secondo dell'età nostra«—ed. Favaro 1890: VIII, 76 and 184.

¹⁷ See Strømholm, "Valerio".

shall not wonder that he does not list Archimeces alone. Yet Archimedes receives special treatment. One work of interest is the brief *Encomium geometriae* which was read in the Academia Platina in Milano in 1535¹⁸. It makes use of the »catalogue of geometers« from Proclos' commentary on *Elements* I »which has just been printed together with the Greek Euclid« (namely by Grynaeus in 1533), but inserts explanations and commentaries¹⁹. Basing himself on Plato's *Phaedo* (where the philosopher Euclid from Megara is mentioned) he inserts Euclid as Plato's contemporary, which gives him occasion to bring in the whole progression of commentators and translators, from Theon, Proclos, Pappos, Marinos and Hypsicles to Campanus and Bartolomeo Zamberti. To Proclos' list he further adds Aristarch, Porus (probably Sporos, as Richard Lorch has pointed out to me), Nicomedes and Menelaos,

all of them Greeks, and writing in Greek, whose writings will have been lost, but testimony of whom by others is alive. But they are, indeed, all defeated by Archimedes of Syracuse, almost all of whose findings we possess. A man of the highest genius, and who will have shown the circumference of the circle pretty closely, and taught by solid geometry how to interpose two lines between two others in continuous proportion. But that has been lost.²⁰

In the *De subtilitate*²¹ from 1550, book XVI ("De scientiis") contains a list of the ten foremost authors,

¹⁸ In Cardano 1663: IV, 440-445. The quotations are taken from p. 443.

¹⁹ One of these should be noted as an apropos to mathematical Platonism, *viz.*, that Cardano finds it appropriate to supplement Proclos' praise of Plato the mathematician by the information that Plato did not follow up his sympathy for mathematics with actual work on the subject.

²⁰ »... omnes Graeci, Graecéque scripserunt, sed quorum tamen monumenta interierint, aliorumque tantum testimonium viuant. Verùm omnes hos vincit Archimedes Syracusius, cuius fermè omnia inuenta habemus: vir summo ingenio, et qui circuli periferiam proximius ostenderit, et solida demonstratione duabus lineis duas interponere continua proportione docuerit: sed hoc periit«.

²¹ In Cardano 1663: III, 352-672. Quotations from p. 607f.

of whom Archimedes is the first, not only because of his works which have now been published [by Tartaglia in 1543] but also because of his mechanics which, as Plutarch relates, shattered the Roman troops time and again ...²².

Archimedes is then followed by Aristotle; Euclid; John [Duns] Scotus; John (sic!) Swineshead the author of the *Liber calculationum*; Apollonios »almost from the same epoch as Archimedes«²³; Archytas; Muhammed ibn Mūsā [al-Khwārizmī], and Jābir [ibn Aflah] of Spain. Number 11 as to subtlety is Galen—and as number 12 comes Vitruvius »who could have been counted among the first if only he had described his own findings and not those of others«. As a commentary to the list it is explained that each is excellent in his own way, Aristotle in genius (»ingenio«), Archimedes in genius and imagination (»imaginatione«), Swineshead in imagination, etc.

Cardano was a tolerant eclectic to whom »every truth is divine«24—yet in view of his own very un-Archimedean approach to mathematics (not to speak of his philosophical and scientific orientation in general and his recent clash with Tartaglia the publisher of Moerbeke's Archimedean translations) it is striking that Archimedes comes in first, both within geometry and in a discussion of the sciences in general (in a list dominated by mathematicians, it is true, but where Cardano's own arithmetical and algebraic interests are given as modest a standing as Galenic medicine). Cardano's Archimedes, even more clearly than Regiomontanus', was clearly not a figure moulded freely to fit and justify Cardano's own doings but an archetype representing a general idea of the nature and role of mathematics.

 $^{^{22}}$ »Archimedes primus sit, non solum ob monumenta illius nunc vulgata, sed ob mechanica, quibus vt Plutarchus auctor est, vires Romanorum saepius fregit ...«.

^{23 »}Sextus locus Apollonio Pergeo debetur, qui fermè aetate aequalis fuit Archimedi«.

²⁴ »Omnis enim veritas diuina est«—De subtilitate, p. 607.

That those who translated Archimedes (Commandino) or purported to have translated him (Tartaglia) had a high opinion of him is hardly astonishing, and proves less than the very fact that they undertook to translate and/or to publish, and I shall discuss neither in detail. Yet before we leave sixteenth century Italy it will be appropriate to look at Baldi's 201 Vite di matematici. The most extensive of these is the Life of Pythagoras²⁵ (64 pages in the autograph), which however leads off with a series of excuses for including Pythagoras (since Thales was included, who inaugurated philosophy in Greece, the princeps of Italian philosophy should be included too; etc.). Obviously, the length of the biography does not reflect Pythagoras' status as a mathematician in the opinion of Baldi's »model reader«. Second in length is the biography of Archimedes, with 51 pages in the autograph (more than the double of any other). It begins in a totally different vein:

In all domains there have been some who, having arrived at the peak of excellence, have demonstrated how far the human intellect could advance in that direction. Without doubt Archimedes was such a man in mathematics, since the first place is due for good reasons to him. That I shall write about him therefore distresses me for several reasons. Namely that my talent is not proportionate to the topic; that the high age of the subject does not allow me to know everything concerning him and worth telling; furthermore the lack of books, and the place where I sojourn, far not only from the famous libraries but from the most tiny ones²⁶,

²⁵ Ed. Narducci 1887.

²⁶ »In tutte le facoltà ui sono stati alcuni, che, ariuati al colmo dell'eccellenza, hanno mostrato quanto in quella possa auanzarsi l'intelletto humano. Tale senza alcun dubbio fù Archimede fra'Matematici, poichè ad esso ragioneuolmente si conuiene il primo luogo; onde, douende scriuere de lui, mi dolerò di più cose, cioè dell'ingegno non proportionato a'meriti del suggetto, dell'antichità, che non me lascia giungere alla cognitione di tutte le sue cose degne d'historia, e l'altra la penuria dei libri, e il luogo oue mi trouo, non solo lontano dalle librarie famose, ma'anco dalle minime« (ed. Narducci 1886: 388).

after which he discusses all aspects of Archimedes' life and accomplishment which a critical mind could read about in a famous library, including that which is found in Archimedes' own works, that which is told by other mathematicians, and those anecdotes about his engineering triumphs which are told by Plutarch and similar writers. In the final paragraph it is concluded that

Archimedes has been the Prince of mathematicians; whence Commandino said with good reason that the one who has not studied Archimedes' works with diligence can hardly call himself a mathematician.²²

These lines were written August 25, 1595, some 250 years after Petrarch and a few contemporaries had instituted Humanist Archimedism in Italy. They, and Baldi's biography in general, demonstrate to what extent Humanist scholarship on Ancient mathematics and mathematical work on Archimedean foundations had grown together while fecundating each other—in Italy.

IV. Northern Humanism

-in Italy-but what had happened elsewhere?

The first observation to make in order to answer this question is that the Humanist movement itself was originally an Italian movement. Petrarch's contemporaries in Paris and Oxford were

²⁷ »È stato Archimede il principe de'matematici; onde con molta ragione diceua il Commandino, a pena potersi chiamare matematico chi con diligenza non haueva studiato l'opere d'Archimede« (ed. Narducci 1886: 453).

precisely those producers of *sophismata* and *calculationes* who were covered with scorn by fifteenth and sixteenth century literary Humanists. There were also those like Jean de Murs, who could be recognized as a good mathematician by Regiomontanus and who anticipated the Renaissance integration of theoretical and practical knowledge as well as the civic utilitarianism of the following century but who no more than Italian mathematicians of the fourteenth century belongs to the Renaissance movement proper.

Then, from the mid-fifteenth century onwards there is a definite revival of mathematical interest in the Provençal-French area somehow related to Italian abacus school mathematics and culminating with Chuquet's works28. As far as I know the surviving writings (primarily Chuquet's works), however, the movement partakes no more in the Archimedist current (or in the assimilation of Greek mathematical writings) than does its Italian counterpart before Luca Pacioli. The best-selling Margarita Philosophica, written by Gregor Reisch in 1496 and printed time and again in French and German territories between 1504 and 1517 (facsimile from Reisch's 1517 edition in Geldsetzer 1973), is equally irrelevant to both the new tendencies in mathematics (in several respects Reisch manages to get below the level of Isidore of Seville's Etymologies!); to Humanism (if any characterization can be given to its messy philosophical eclecticism it is diluted Medieval Aristotelianism); and to Archimedism (Archimedes is a non-person)29.

With a few exceptions (Lefèvre d'Étaples and Melanchton being the most important—exceptions to whom I shall return) Northern Humanists excluded Ancient mathematics and technology from their field of interest until c. 1550. Mathematics, it is true, or at least the

²⁸ A discussion of the whole wave, from the mid-fifteenth century beginnings to its disappearance with Estienne de la Roche in the 1520es, is in van Egmond 1988.

²⁹ There is no reason to substantiate these claims in this place. I can refer to my (1987: 60-63).

science of numbers, was not unknown to them. Reuchlin's and Agrippa's mathematical interest, however, were kabalistic and magical. Even in connections where an Italian Humanist would normally have referred to Archimedes (viz. in connection with combined mirrors, automata and similar astonishing technologies) Agrippa's De occulta philosophia (1533: lxxxix-ci) leaves him unmentioned. Thomas More refers to the excellency of the Utopians in »musica [...] ac numerandi et metiendi scientia« (ed. Surtz & Hexter 1965: 158) but omits both Euclid and Archimedes from his list of important Greek books (pp. 180-182) in a way which stuns his commentators (p. 435) but which is in fact characteristic: More was a good Renaissance utilitarian, as documented in many ways in his Utopia, and he recognized the importance of basic mathematics (»counting and measuring ...«). But since mathematics was not part of his picture of Ancient culture his mathematics remained at this utterly elementary level. In this respect he did not differ from his friend Erasmus nor from most other Northern Humanists³⁰.

The first exception of some importance was Jacques Lefèvre d'Étaples (c. 1455-1536). He was a sincere Neo-Platonist and had been to Italy; there he had been in touch with Ficino, Ermolao Barbaro and Pico della Mirandola, and he was inspired by his knowledge of Italian philosophy to elevate the mathematical level of French learning³¹. Lefèvre undertook a number of mathematical publications, among which were two editions of Jordanus' *Arithmetica* (1494 and 1514); yet precisely this acme shows the »Medievally

³⁰ Erasmus, it is true, knew that »references to music, geometry, arithmetic, astronomy, medicine, and if you like to add it, magic too, occur not infrequently in medicine«; this, on the other hand, is his only reason to argue for some teaching of »arithmetic, music, and astronomy«. Still, these subjects »need only be sampled«, i.e., be treated on the level of Isidore of Sevilla (*De recta pronuntiatione*, tran. M. Pope, *in* E. K. Sowards (ed.) 1985: 371, 387).

³¹ Lefèvre d'Étaples is thus also an exception to the rule that Ficinian Neo-Platonism was not conducive to mathematical activity.

solid« character of the venture—the most modern piece being perhaps the least solid, viz. Charles Bouvelles squaring of the circle etc. (1503). The paragon of French mathematical publishing around 1500 had learned of the importance of mathematics through his acquaintance with Italian Humanism; but his idea of mathematics (or the idea which could be expected to gain acceptance in Paris) was traditional, far from any Renascence of Ancient mathematical writings or style. Far, indeed, from anything reminding of Archimedism.

The other main exception is Luther's follower Melanchton (1497-1560). While Lefèvre had written at least an introduction to Boethius' Arithmetic and a few similar things, Melanchton was no mathematical author at all. But he lent his name and fame to many a mathematical book as a writer of prefaces. One of these (written 1537) is found in the Basel edition of Campanus' and Zamberti's translations of the Elements (Euclidis megarensis ... 1546, fol. *2-*4). Here Melanchton shows that he is very well versed in what Plato and Aristotle write about mathematics-but especially in their opinions on the moral implications of mathematics (the central point being the identification of Justice with geometrical proportionality). These moral teachings are Alpha as well as Omega to Melanchton. In a curious way, mathematics is first moral philosophy. But this role is, of course, totally alien to all those ideas which could be attached to the figure of Archimedes: The wondrous technician, the master of civic utility, the Prince of pure geometry. Melanchton, together with Lefèvre, represented a new strain in Northern Humanism, viz. the opening up of Humanist thought to mathematics. But because Northern Humanism had another perspective on Ancient thought and culture (and, we might say, a more narrow, literary and moralizing perspective---»Northern Antiquity« was dug out from libraries and written on parchment, while »Italian Antiquity« was full-bodied and everywhere present though in ruins) its way toward this opening to mathematics was different from that of Italian Humanism. Archimedism was beyond its horizon, the best it could aim at for the moment was Euclideanism.

Eventually, however, Archimedism did penetrate Northern Humanism. In contrast to the development in Italy, where glorification had preceded understanding of the mathematical works, Northern Humanism only assigned paradigmatic status to Archimedes when it had grasped at least vaguely what he stood for in mathematics.

The beginning was made in France. In the generation after Lefèvre, the paramount French mathematician was Oronce Fine (1494-1555), who was certainly not of a mathematical stature to grasp however vaguely. In those of his works which I have looked at there is, correspondingly, no real trace of Archimedism³², nor is any hint in this direction to be found in his vast bibliography (Hillard & Poulle 1971; R. P. Ross 1974).

Even Fine's student Petrus Ramus (1515-1572), of course, was no mathematical genius. But it was Ramus who, in this the first of Northern Humanists, knew both classical literature and mathematics well enough to inaugurate the era of French Archimedism—not least, perhaps, because he was also enough of an ideologue to disregard disturbing facts.

All this is clearly borne out by the Scholarum mathematicarum libri unus et triginta (1569). Book I (pp. 1-40) deals with "the first inventors and authors of mathematics" (p. 41), from Adam to Theon and Proclos. Already the table of contents (p. β 3) is striking: no other author is dealt with on more than two consecutive pages; Archimedes occupies seven (pp. 26-33). A look at the text shows,

³² In his *Protomathesis* (1532) a reference to Archimedes *will* be found, and even an Archimedean proof: fols 85° to 89° contain an Archimedean proof that the ratio between the circular periphery and the diameter lies between $3^{10}/_{71}$ and $3^{1}/_{7}$, and tells that the proof was found by Archimedes "according to common assumption" ("siuxta vulgatum"). Alas, this correct proof (yet very Medieval approach) is followed by an "exact" construction of Fine's own making (fols. 89° to 91°).

firstly, that Ramus was just as erudite as Baldi, though certainly less modest; secondly that Ramus impressed his own utilitarianism on the material, censuring Plato and Euclid because of their unmistakable alignment of mathematics with philosophy or pure knowledge and weighing the evidence on Archimedes according to its agreement with his own predilections; thirdly, that Archimedes was, to Ramus no less than to Baldi, the Prince of mathematicians. The account begins with almost the same *topos*:

If God had decided that there should be in each art something like a unique idea which everybody studying the discipline would propose to himself as a model—as in eloquence, Demosthenes and Cicero, and in medicine Hippocrates and Galen—then it would be Archimedes in mathematics³³.

Ramus then tells about Archimedes' excellence in all mathematical disciplines, which is sufficient to outweigh the "obscurity of his method and whole way to present the matter", and which even makes Ramus accept his infinitesimal investigations (otherwise hardly an interest in agreement with Ramus' utilitarian canon) (p. 26). On the testimony of Plutarch it is admitted that Archimedes may have been imbued with the "error of Plato", but from other biographical anecdotes it is argued that he must at least have been much less so than Euclid (p. 28)—and most of the treatment from then on deals with Archimedes' technical feats and the sources ascribing technical writings to the great man, interrupted (p. 30) by another statement that Plato's "blind ambition" not only spoiled the applicability of geometry but virtually the science itself, causing the subsequent 1500 years to bring nothing new to geometry.

In the final page of his history of Ancient mathematics Ramus concludes that

^{33 »}Voluit deus in omnibus artibus aliquam velut ideam singularem esse, quam omnes ejus disciplinae studiosi ad imitandum sibi proponerent: ut in eloquentia, Demosthenem et Ciceronem: in medicina Hippocratem et Galenum: sic in mathematicis Archimedem« (Ramus 1569: 26).

If the composition and arrangement of mathematical teaching be looked for, Hippocrates, Leon, Theudios, Hermotimos, Euclid and Theon will carry off the first fruits of praise as authors of *Elements*; if the nobility and breadth of mathematical leisurely studies be assessed, Pythagoras, Plato and Aristotle will deserve authority in mathematics. But if not only the scholastic truth and proofs from books but also its public use and its utility be appraised, which indeed is the most valuable, Archytas, Eudoxos, Eratosthenes, but, greatest and towering over everybody, Archimedes alone is to be elevated to the skies.²⁴

More lavish praise could hardly be imagined; formulated by an author whose natural bent was toward the engineer but whose competence was sufficient to see the real greatness of Archimedes in geometry it shows that not only veneration for Archimedes but fullyfledged Archimedism had now entered French mathematics—namely the acceptance of Archimedes as »an archetype representing a general idea of the nature and role of mathematics«, as Cardano's attitude was formulated above. Now as always, of course, this archetype was by necessity identical with the understanding of the historical Archimedes which was held at the time, and no transhistorical entity. But even if historically conditioned it could not be twisted at will; commitment to the Archimedist ideal was a constraint, and the commitment to Ancient mathematics inherent in the ideal will have been important in blinding Ramus to the agreement between his utilitarian ideals and the organization of much of the Islamic mathematical heritage.

³⁴ »Nam si mathematicae institutionis compositio et conformatio spectetur, στοιχειοται Hippocrates, Leo, Theudius, Hermotimus, Euclides, Theon principem fructum laudis ferent, si nobilitas mathematicae scholae et amplitudo perpendatur, mathematum authoritas ad Pythagoram, Platonem, Aristotelem pertinebit: si, quod summum est, mathematum non solum scolastica veritas et é libris demonstratio, sed popularis usus atque utilitas aestimatur, Archytas, Eudoxus, Eratosthenes, sed maximé et altissimé supra omnes unus Archimedes in caelum ferendus erit« (Ramus 1569: 40).

In itself this blindness was of course no gain for Ramus' mathematics, as demonstrated by his Algebra35. It is typically Ramist in its way to set up schemes for calculation with algebraic expressions, and tries to assimilate the subject to Ancient mathematics through a definition of algebra as »a part of arithmetic, which from imagined continued proportions establishes a certain form of counting of its own«36; i.e., it assimilates the sequence of algebraic powers unitas (x^0) , latus (x^1) , quadratus (x^2) , etc. with a continued proportion—a concept which was familiar from elementary Euclidean mathematics but which offered no really new insights into the nature of algebra. But apart from the not very useful schemes (presented as mere rules without proof) and the use of abbreviations (u for unitas, l for latus, q for quadratum, c for cubum, bq for biquadratum, etc.) nothing surpassing the level of al-Khwārizmī's original treatise or its twelfth-century Latin translations will be found. In this particular field, the constraints presented by Ramus' Archimedism and his love of schemes made him change the formal dress of the subject—but they did not help creating a break-through.

That was not the fault of the basic Archimedist idea, and another French Archimedist—less outspoken ideologically but of greater mathematical perspicacity—was able to bring the idea to fruition. François Viète (1540-1603), indeed, was no less sure than Ramus that what had happened to mathematics since the end of the Ancient era was better not counted at all³⁷. In the dedicatory letter of the *In artem*

³⁵ Ramus 1560. I take advantage of this occasion to point out that the work is indeed published anonymously, and not under Ramus' name as I have claimed earlier. As discovered by Warren van Egmond the author's name found in the copy in the University Library in Copenhagen has been inserted in ink, in a way which couldn't be distinguished from print in the microfilm copy at my disposal, but which aroused van Egmond's suspicion because it was not centered as the rest of the title page.

³⁶ »Algebra est pars Arithmeticae, quae é figuratis continué proportionalibus numerationem quandam propriam instituit« (Ramus 1560:A ij).

³⁷ In a more literal way, too, Viète can be counted an Archimedist. He is more parsimonious than Ramus in the discussion of authors; but if references can make it out

analyticam isagoge he explained that existing algebra was »so defiled and polluted by barbarians« that he found it necessary »to bring it into, and invent, a completely new form«38. This new form, inspired by Pappos' discussion of the concept of analysis (making possible the formulation of the theory and of the metatheoretical status of algebra) and the stringent distinction between quantities of different kind (required by Aristotelian philosophy and entailing the principle of homogeneity) was, as we know, the starting point of Modern algebra (or, as some authors would have it, the starting point of algebra as distinct from a mere algebraic approach—see Mahoney 1971: 372).

V. The new philosophies

Further illustrations of a sudden but diverse impact of Archimedism in late sixteenth century France and England would follow if, e.g., Foix de Candale and John Dee were scrutinized. I shall abstain from that, and close my investigation by suggesting how the Archimedist ideology was transformed and absorbed into the hegemonic ideas of mid-seventeenth century philosophy.

for explicit praise it can be observed in the index to the collected works that Archimedes turns up 13 times while Euclid and Apollonios must content themselves with 7 references each (and Plato with 4 and Aristotle with 2). See Hofmann 1970: XXXII'-XXXIX'.

³⁸ »Ecce ars quam profero nova est, aut demùm ita vetusta, et à barbaris defoedata et conspurcata, ut novam omninò formam ei inducere et [...] excogitare necesse habuerim« (ed. Hofmann 1970: XI).

This transformation can, e.g., be illustrated by Marin Getaldić's Promotus Archimedis seu De variis corporum generibus gravitate et magnitudine comparatis from 1603 (facsimile edition in Dadić 1968: 1-80). The work, dealing with compared specific gravities, is composed win the geometrical manner«, as it would be called later on, from propositions (some of them "theorems«, others "problems«) with enunciation and proof and interspersed "examples«. The style could of course be considered Euclidean. But it was Archimedes, not Euclid, who was known to have used this style to describe physical problems, and the reference to Archimedes in the title of the work is therefore most apt. Still, the construction more geometrico is only one aspect of the work; another aspect quite as important is the experimental foundation provided by actual measurement.

Experimenta was no new idea in Late Medieval and Renaissance philosophy. Originally, however, the term was seen in opposition to stringent demonstration³⁹, and even in the sixteenth century experimentation was normally not quantified nor mathematicized (optics, one of the classical »intermediate sciences, being an exception). The introduction of quantified experiment by Getaldić and his contemporaries is, significantly, referred to Archimedes and is, indeed, a result of the synthesis of the two aspects of the traditional dichotomized Archimedes: the stringent and ingenious geometer and the wondrous mathematical engineer.

Getaldic's work is thus, firstly, a portent of the spread of the »geometrical method« to a great many branches of seventeenth century philosophy, and evidence that the geometer inspiring that method is Archimedes rather than Euclid (even though this distinction will of course have been blurred when we arrive at the epoch of a Spinoza). (Investigation of the Galilean corpus would

³⁹ So in Richard de Fournival's *Biblionomia*, where Jordanus de Nemore's *apodixeis* (i.e., works based on rigorous mathematical proof) were opposed to purely empirical *experimenta* on algebra or progressions—e.g. in N° 45 and 48 (ed. Birkenmajer 1970/1922:

point in the same direction; cf. C. Dollo's contribution to the present volume).

Secondly, it points to the importance of the Archimedist ideology for the formation of seventeenth century "experimental philosophy". For a movement inspired by utilitarians and sceptical empiricists like Bacon and Ramus it lay near at hand to dismiss mathematical exactitude as irrelevant (as indeed often done, e.g., in chemistry until Lavoisier). But as long as Archimedes served as a "culture hero" it lay equally near at hand for those inclined toward mathematization and exactitude to make appeal to his authority, while it was correspondingly difficult for others to dismiss the appeal. This was done time and again by Galileo. On the other hand the appeal to Archimedes could also be used to legitimate the use of empirical data against any a-prioristic opposition (be it traditionalistic-Aristotelian or rationalistic-metaphysical). This was done no less often by Galileo, but also, e.g., by Descartes in a letter to Mersenne:

You ask me whether I think what I have written about refraction is a demonstration. I think it is, at least as far as it is possible, without having proved the principles of physics previously by metaphysics [...]. But to demand that I should give geometrical demonstrations of matters which depend on physics is to demand that I should do the impossible. If you restrict the use of "demonstration" to geometrical proofs only, you will be obliged to say that Archimedes demonstrated nothing in mechanics, nor Vitellio in optics, nor Ptolemy in astronomy, etc. 40

The historical Archimedes was, no doubt, an extraordinary mathematician and probably an extraordinary technician, too. Still, he could of course not avoid being a man of his times, answering the questions of this time, even formulating them, but still formulating questions which had meaning in their own historical context. The »Archimedes« of the Renaissance, from Petrarch to Getaldić, Galileo and Descartes, was a man of his times, i.e., of the fourteenth to the

Letter to Mersenne, 11. October 1638, trans. Crombie, "Descartes", p.53.

seventeenth century. He changed with these, in particular with the advances in mathematical and classical scholarship. But due to the advances in scholarship *Archimedism*, that mathematicians' ideology which brandished the "Archimedes" of the time as "something like a unique idea which everybody studying the discipline would propose to himself as a model" (to quote Ramus), carried decisive messages from Antiquity to Renaissance and Early Modern Europe. Here they went into the synthesis which gave rise to Modern science and at least to a certain moment of Modern philosophy—and, mediated by Hume's and Kant's and the Romanticists' reactions, indeed to any Modern philosophy. In this way, Archimedes as well as "Archimedes" may be more with us today than we usually notice. Of course, Plato is so, too—but *not* because of influence in the formation of Modern mathematics.

VI. Appendix

On the whole, the career of Archimedism ended when it was absorbed into Early Modern philosophy. None the less, a slightly belated formulation of the praise of Archimedes deserves to be quoted, because it combines archaic characteristics—viz. the accumulation of all the delightful Ancient anecdotes—with new attitudes pointing forward toward nineteenth-century Neo-Humanism and toward science-based imperialism. The text in question is an introductory poem from the first German translation of Archimedes from 1670, in which

Der in Teutschland wieder lebende und die Teutschen zu Höherachtung derer mathematischen Wissenschaften ermahnende Archimedes

explains the importance of his discipline as follows:

Ich weiß, O Teutsche Welt, daß unser hohes Wissen, (Die Meß-, Beweg- und Waag-, die Bau- und Sternenkunst) Bey dir hat lange Zeit verachtet ligen müssen mit andern Künsten nie genossen gleiche Gunst! War Dürer schon bemüht, das Werk belobt zu machen: fand Apianens Kunst gleich bey dem Kayser Gnad, und Brahe Königs-Huld: ob andrer hohe Sachen die teutsche Vorder-Welt wol eh geliebet hat: So hat doch dieser Zeit die Kunst die Gunst verloren. und muß, dem Sprichwort nach, nach Brod und Betteln gehn; Bey sonderen Gestirn muß jezund seyn gebohren, wem die Gedanken heut nach diesen Künsten stehn! Dem schlechten Pöfel wird, als eigen zugeschrieben was weiland Keyserlich, was Fürstlich, Herrlich war: Was Atlas, Julius, was Könige getrieben des schämt der Adel sich, das hasst der Lehrer Schaar; Und bey den Teutschen nur! Ihr Edle Teutsche Sinnen, erhebt was euch erhebt! liebt diesen Tugend-Schein, Die Künste, die den Stand berühmter machen können! Ich selbsten kan euch deß ein klares Beyspiel seyn. Sizilien hat mich, das reichste Land, gezeuget; der Haupt- und Königs-Sitz ist meine Vatter-Stadt. Mich hat ein' edle Brust, von hohem Stand, gesäuget, die Brust, so Könige zu Blutsverwandten hat, doch kundt der Königs-Nahm sich nicht unsterblich machen, die Künste brachten Ihm des Immerlebens Liecht, Mein Wissen riß ihn erst aus des Vergessens Rachen.

Todt wäre Hieron, lebt' Archimedes nicht. Was würde wol die Welt von Gelons Krone wissen, die er aus klarem Gold den Göttern machen Hieß; Wann nicht des Meisters List der meinen weichen müssen, so daß sich der Betrug nicht länger bergen ließ? Wer hätte, Syrakus, du Vatters-Stadt, beschrieben, wie schwär du deinen Fall der Römer Macht gemacht? Der Ruhm der Dapferkeit ist dir allein geblieben, durch das, was meine Kunst zu wegen hat gebracht, Die Centner-Schleuderstein, die ich den Feinden schenkte, Der Pfeile Hagelgschoß, die grimme Eisenhand, Die manches Krieges-Schiff, mit leichter Müh versenkte, die machten, O Marcell, dir unsre Macht bekannt; Dir und dem grossen Rom, das meine Faust gelehret was solcher Künste Kraft, wir groß, wie nützlich sey. Diß lerne Teutschland auch, wann es dergleichen höret, und glaube ferner nicht dem falschen Luftgeschrey, So sie unnützlich nennt: Es frag' in andern Landen wie mancher Sieg im Krieg, wie mancher Stadt Verlust, wie reicher Schätze Raub, auf diesen Fuß gestanden: Der Schad lehrt manchen erst, was er sonst nicht gewust: Doch nicht dem Krieg allein dient diß gelehrte Wissen, des Baumes süsse Frucht hat auch der Fried geschmekkt: Oft hat sich über sich die Witz verwundern müssen, wann, was kaum menschlich war, des Menschen Sinn entdekkt. Wann diese meine Hand durch meine Schnekkenwinde was Tausend nicht vermocht, wom Land ins Meer gebracht: Wann meine Wasserchraub die Tiefsten Sümpf und Gründe Aegyptens ausgeschöpft: Wann eines Mannes Macht viel Tausend Scheffel Korn in freye Luft gehoben: Wann durch ein rundes Glaß ein Wunderbild der Welt des ganzen Himmels Lauf, was unten geht und oben, kunstrichtig allerseits für Augen war gestellt.

Diß und noch anders mehr kunnt Hieron bewegen zu glauben was forthin sein Archimedes wolt; Wann Er auch selbst die Erd in ihrem Punct zu regen nach festgegebnen Stand sich unterfangen solt. Auch in des Feindes Herz hat Lieb für Rach geboren so grosser Thaten Ruhm: für dieses graue Haar, Für dieses Haupt, daß ich, nach Feindes Recht, verlohren, ward ein Verbot gesetzt der grimmen Krieger-Schaar. Die Wut verschonte mein: Die Kunst nahm mir das Leben, die, so das Leben mir und meinen Freunden gab. O süsse Wissens-Lust, darinn ich pflag zu schweben, da ich mich sterbend grub im Sand mein eignes Grab; Da ich, mir selbst entrükkt, mich mit Gedanken speiste und ohn' Empfindlichkeit in tiefstem Denken saß; Da gleichsam aus dem Leib die freye Seele reiste, und Essens, Trinkens, ja des Lebens gar, vergaß. Jedoch, wie kunnt ich auch des Lebens wol vergessen, weil Denken Leben ist, und ich auch sterbend dacht? Hat schon die Spitze mich zu tödten sich vermessen, doch hat sie nur den Leib, die Seel nicht umbgebracht. Mein Geist ist jezund noch, mein Ruhm ist nie gestorben. Sizilien mich begrub; in Teutschland steh' ich auf. Die Grabschrift zeiget dort, was ich durch Kunst erworben, in 3 und 2 besteht mein ganzer Lebens-Lauf. Recht leben, Wissen ist: Das, was wir sonst beginnen ohn alle Wissenschaft, ist auch dem Vieh gemein; Vernunft die Menschheit macht; in Denken, Wissen, Sinnen, nicht in der Nutzbarkeit, will Menschen-Wesen seyn. Preist schon der Eigennutz die Werke meiner Hände; Ich selbsten achte mehr was bloß im Denken steht; Des Kreisses Vierung ich mit mehrer Freude fände, als alles das, wordurch ich jene Wunder thät. Wolan du Teutscher Sinn! Steht deine Lust im Sinnen,

so suche deinem Geist dergleichen Wissenschaft!

Was recht vernünftig macht, was geistig, ist hierinnen,
Hier wird dem Menschen-Seyn Vollkommenheit geschafft.

Ist dann dein Wissen, nur mit Wissen, nicht vergnüget,
und suchet dein Gemüht des Wissens Nutzbarkeit?

So glaube dem, der dir [frag' Hieron] nicht lüget:
Hier ist der gröste Nutz der ganzen Welt bereit.

Was Büchs und Schwerdt gewinnt, was Schiff und Segel bringen
der Indien reiche Schätz, was Teutschland reicher macht!

Die ganze andre Welt mit ihren feinsten Dingen
hat dieser Künste Fleiß in unsre Welt gebracht.41

⁴¹ Des unvergleichlichen Archimedis Kunst-Bücher oder Heutigs Tags befindliche Schriften, aus dem Griechischen in das Hoch-Teutsche übersetzt, und mit nohtwendigen Anmerkungen durch und durch erläutert von Johanne Christophoro Sturmio. Nürnberg / In Verlegung Paulus Fürstens, Kunst- und Buchhändlers Seel. Wittib und Erben, 1670.

The poem takes up a bit more than two folio page. It is followed by 5 pages of notes, explaining everything from the favours bestowed upon Peter Apian by the Emperor to the sources for the various anecdotes on Archimedes.

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Author's Address:

Jens Høyrup
Institute of Communication Research,
Educational Research
and Theory of Science
Roskilde University Centre
P. O. Box 260
DK-4000 Roskilde
Denmark

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