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SELEUCID INNOVATIONS IN THE BABYLONIAN "ALGEBRAIC" TRADITION AND THEIR KIN ABROAD

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The surveyors' tradition and its impact in Mesopotamia

Much of my work during the last decade or so has dealt with what I have called the "surveyors' tradition" and its impact on Babylonian, Greek and Islamic "literate" mathematics. Most of the results have been published elsewhere,[1] for which reason I shall restrict this introduction to what is crucial for the theme I have been asked to deal with.

The tradition in question is a "lay", that is, non-scribal and for long certainly also in the main an oral tradition (barring a single known exception, non-scribal literacy only became a possibility with the advent of alphabetic writing); elsewhere, when discussing the characteristics of such traditions, I have termed them "sub-scientific".[2] Its early traces are found in the Mesopotamian literate record; it is plausible but not supported by any positive evidence that it was also present in the Syrian and the core Iranian regions in the second millennium BCE; influence in Egypt at that date is unlikely.[3] In the later first millennium BCE it influenced both Demotic Egypt and Greek and Hellenistic mathematics. Whether the altar geometry of the Śulvasūtras was also influenced is less easily decided (quite apart from the question whether the constructions described in these first millennium writings go back to early Vedic, that is, second-millennium practice); however, the indubitable traces of the tradition in Mahāvīra's 9th century CE *Gaṇita-sāra-sangraha* are likely to go back to early Jaina times and thus to the late first millennium BCE or the first centuries CE (see below). Even though the intermediaries are not identified, the

[1] A concise presentation will be found in [Høyrup 1997d], a fuller discussion in [Høyrup 1998] – both concentrating on the proto-algebraic aspects of its material. Aspects related to practical geometry proper are dealt with in [Høyrup 1997a].

[2] See [Høyrup 1990a], [Høyrup 1997c] and [Høyrup 1997b].

[3] Firstly, neither the Rhind Mathematical Papyrus nor the Moscow Mathematical Papyrus betrays any acquaintance with it. Secondly, one problem group in the Rhind Papyrus (the filling problems, no. 35–38) turns out to be related to (for linguistic reasons indeed to be derived from) West Asian practitioners' mathematics (see [Høyrup 1999a: 124]), indicating that such borrowed problems were not *a priori* ostracized, which would otherwise be a possible explanation of their absence from the two papyri; but this problem type has nothing to do with mensuration. Thirdly, the problems pointing to West Asia that turn up in Demotic sources are exclusively related to the innovations of the Seleucid age (see below).

Syrian, Iranian and Central Asian regions are unlikely not to have been involved. Possible influence on the Chinese *Nine Chapters on Arithmetic* is a question to which we shall return.

Many traces of the tradition are found in Arabic sources from the later first and early second millennium CE; some turn up, finally, in Italian *abbaco* writings from the late Middle Ages – in part derived from earlier Latin translations of classical Arabic works, in part however from what must have been still living tradition in the Islamic world.

One of the Arabic works – perhaps written around 800 CE, perhaps only a witness of the terminology and thus of the situation of that period, immediately preceding the work of al-Khwārizmī – is a *Liber mensurationum* written by one Abū Bakr, so far known only in a Latin translation due to Gherardo da Cremona[4] (whence the Latin title). Strangely, it still conserves much of the characteristic phraseology already found in Old Babylonian (but not late Babylonian) problems derived from the tradition[5] – so much indeed that transmission within some kind of institutionalized teaching network seems plausible, perhaps supported by writing since the advent of non-scribal Aramaic alphabetic literacy. Even though we know nothing about the geographic origins of Abū Bakr, this consideration seems to locate him (or an earlier written source which inspired him) in the Iraqo-Syrian area – comparison with other descendants of the tradition shows that none of these conserves the original formulations as faithfully as Abū Bakr.

Basic applied mathematics is often so unspecific that shared methods constitute no argument in favour of transmission or borrowing. Once area measures are based on the square on the length unit, there is only one acceptable way to find the area of a rectangle, and only one intuitively obvious way to approximate the area of an almost-rectangular area (*viz* the "surveyors' formula", average length times average width). Certain geometric procedures are less constrained by the subject-matter and may serve, but the best evidence for links is normally supplied by recreational

[4] Ed. [Busard 1968]. Abū Bakr's use of the terms *al-jabr* and *al-muqābalah* is pre-al-Khwārizmīan (cf. [Saliba 1972] on the changing use of the two terms); moreover, *murabbaʿ* is used exclusively in the sense of an unspecified "quadrangle" and speaks of a square as an "equilateral and equiangular *murabbaʿ*", which is another archaic feature. Abū Bakr may still have written at a later moment, but if so as an exponent of a tradition whose roots have not been much affected by the innovations brought about by al-Khwārizmī and Abū Kāmil.

[5] See [Høyrup 1986].

problems, those mathematical riddles that in pre-modern mathematical practical professions served to confirm professional identity.

The main evidence for the existence and long-term survival of the surveyors' tradition is indeed provided by a set of geometrical riddles that turn up in all the contexts mentioned above, in formulations that often exclude mere transmission within the literate traditions and from one literate tradition (as known to us, at least) to another.

The original core of this set of riddles contained a group dealing with a single square ("???" indicates doubt as to the date from which the problem was present; s designates the side, ${}_4s$ "all four sides", d the diagonal and Q the area of the square; here and everywhere in the following, Greek letters stand for given numbers):

$s+Q = \alpha$ (= 110)

${}_4s+Q = \alpha$ (= 140)

$Q–s = \alpha$

$Q–{}_4s = \alpha$ (???)

$s–Q = \alpha$

${}_4s–Q = \alpha$ (???)

${}_4s = Q$

$d–s = 4$ (???)[6]

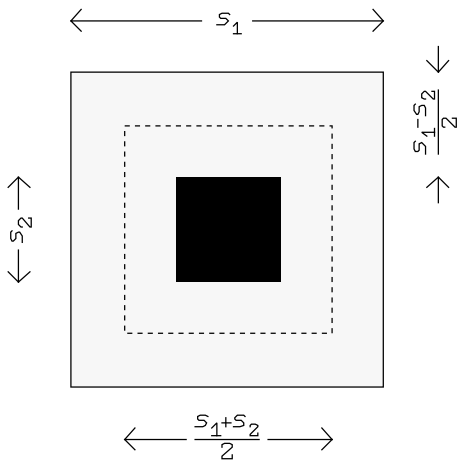

Figure 1. Illustration of the principle that the area of a quadratic border equals ⊏⊐(*l*,*w*), where *l* is its "mid-length" and *w* its width.

Other early riddles will have treated of two squares:

$Q_1+Q_2 = \alpha,\ s_1\pm s_2 = \beta$

$Q_1–Q_2 = \alpha,\ s_1\pm s_2 = \beta$

When the difference between the areas is given, the two squares were thought of as concentric and the difference thus as the area of a quadratic border; at least in later (classical and medieval) times the areas of such quadratic and circular borders were determined as the product of the "average length" (in the quadratic case $2s_1+2s_2$) and the width (in the quadratic case $(s_1–s_2)/2$) – cf. Figure 1. It would be very strange if the same intuitively evident rule (a "naive" version of *Elements* II.8) were not used in early times.

[6] The problem $d–s = 4$ turns up in the *Liber mensurationum* in a way which shows it to be somehow traditional; evidently it refers to the practical assumption that d = 14 if s = 10. But the evidence is insufficient to prove that this was a traditional *riddle* – even a traditional *approximation* might inspire a problem once it was discovered how to solve it correctly.

These problems dealing with a rectangle (length l, width w, diagonal d, area A) will have circulated in the earliest second millennium:[7]

$$A = \alpha,\ l \pm w = \beta$$

$$A+(l\pm w) = \alpha,\ l \mp w = \beta$$

$$A = \alpha,\ d = \beta$$

On circles (with diameter d, perimeter p and area A), at least the problem

$$d+p+A = \alpha$$

will have circulated at an early date, probably also the simpler version where A is omitted.

Most of the problems will have been solved in the original environment in the same way as in the Old Babylonian school, *viz* by means of a "naive" (that is, non-critical though reasoned) analytical cut-and-paste technique. When a length and a width were involved, the solution made use of what we shall call average ($\mu = \frac{l+w}{2}$) and deviation ($\delta = \frac{l-w}{2}$) – a technique that is familiar from *Elements* II.5–6.

This restricted set was adopted into the Old Babylonian school not long before 1800 BCE, and developed into a genuine mathematical discipline of algebraic character, in which measurable segments served to represent entities of other kinds (at times areas or volumes), and where coefficients were varied freely. But this discipline died with the school institution itself at the collapse of the Old Babylonian social structure around 1600 BCE, and only one trace of its sophistication can be found in later sources (a problem type where the sides of a rectangle represent *igûm* and *igibûm*, "the reciprocal and its reciprocal", a pair of numbers belonging together in the table of

[7] In fact, the two problems $A = \alpha$, $l\pm w = \beta$ are likely to constitute the very beginning together with the problems $A = \alpha$, $l = \beta$ and $A = \alpha$, $w = \beta$. These four constitute a cluster and still go together in ibn Thabāt's *Reckoner's Wealth* from c. 1200 CE) [ed., trans. Rebstock 1993: 124]. The problems $A = \alpha$, $l = \beta$ and $A = \alpha$, $w = \beta$ are already found in Akkadian tablets from the 22nd century BCE, but the other two not yet (which is part of the evidence that the trick of the quadratic completion was only discovered somewhere between 2100 BCE and 1900 BCE).

One may add that area metrologies were sufficiently complex to make the apparently innocuous "division problems" $A = \alpha$, $l = \beta$ and $A = \alpha$, $w = \beta$ unpleasantly complex.

reciprocals). When “algebraic” problems turn up sparingly in Late Babylonian but apparently pre-Seleucid texts (maybe c. 500 BCE?), they appear to have been a fresh borrowing from the surveyors’ tradition,[8] and do not go beyond the restricted range of original riddles.[9] Methods are still traditional, based on naive cut-and-paste geometry and on average and deviation.

Seleucid innovations

Innovation, instead, is unmistakeable in the two Seleucid tablets AO 6484 and BM 34568.[10] The former of these is from the early 2nd century BCE[11], the latter – and most interesting – is undated but roughly contemporary.

One problem from the latter text treats of alligation, all the others deal with rectangular sides, diagonals and areas;[12] apart from determinations of d or A from l and w or of w from d and l, everything is new in some way. Two problems are traditional as such, giving A and either $l+w$ or $l–w$; but the procedures differ from traditional ways, finding for instance in the former case $l–w$ as $\sqrt{(l+w)^2-4A}$, next w as $\frac{(l+w)-(l-w)}{2}$, and finally l as $(l+w)–w$; the method of average and deviation is nowhere used (not even in the reverse version where l and w are found from $l+w$ and $l–w$ as average and deviation).

The remaining problems belong to totally new types. In total, the content of the tablet is as follows:[13]

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[8] In any case, the transmission has not been carried by the Babylonian scribal tradition proper, since the technical use of Sumerograms in the terminology is discontinuous. But we cannot exclude that part of the transmission had been carried by peripheral scribal groups (Hittite or Syrian), as was the case for astrology, cf. [Farber 1993: 253*f*].

[9] See [Friberg 1997]. For supplementary (linguistic) evidence that the texts in question antedate the Seleucid era, see [Høyrup 1999a: 161].

[10] Ed. Neugebauer in [MKT I, 96–99] and Waschow in [MKT III, 14–17], respectively.

[11] See [Høyrup 1990b: 347 n.180].

[12] One is dressed as a problem about a reed leaned against a wall. In the general context of rectangular problems it is obvious, however, that the underlying problem is $d–l = 3$, $w = 9$ (symbols as above).

[13] I interpret the sexagesimal place value numbers in the lowest possible integer order of magnitude and transcribe correspondingly into Arabic numerals.

(1) $l = 4$, $w = 3$; d is found as $½l+w$ – first formulated as a general rule, next done on the actual example.

(2) $l = 4$, $d = 5$; w is found as $\sqrt{d^2-l^2}$.

(3) $d+l = 9$, $w = 3$; l is found as $\frac{½\cdot([d+l]^2-w^2)}{d+l}$, d as $(d+l)-l$.

(4) $d+w = 8$, $l = 4$; solution corresponding to (3).

(5) $l = 60$, $w = 32$; d is found as $\sqrt{l^2+w^2} = 68$.

(6) $l = 60$, $w = 32$; A is found as $l\cdot w$.

(7) $l = 60$, $w = 25$; d is found as $\sqrt{l^2+w^2} = 65$.

(8) $l = 60$, $w = 25$; A is found as $l\cdot w$.

(9) $l+w = 14$, $A = 48$; $\langle l-w\rangle$ is found as $\sqrt{(l+w)^2-4A} = 2$, w as $½\cdot([l+w]-\langle l-w\rangle)$ and l finally as $w+\langle l-w\rangle$.

(10) $l+w = 23$, $d = 17$; $\langle 2A\rangle$ is found as $([l+w]^2-d^2) = 240$, $\langle l-w\rangle$ next as $\sqrt{(l+w)^2-4A} = 7$ – whence l and w follow as in (9).

(11) $d+l = 50$, $w = 20$; solved as (3),[14] $l = 21$, $d = 29$.

(12) $d-l = 3$, $w = 9$ (the reed problem translated into a rectangle problem, cf. note 12); d is found as $\frac{½\cdot(w^2+[d-l]^2)}{d-l} = 15$, l as $\sqrt{d^2-w^2} = 12$.

(13) $d+l = 9$, $d+w = 8$; $\langle l+w+d\rangle$ is found as $\sqrt{(d+l)^2+(d+w)^2-1} = 12$, where 1 obviously stands for $(l-w)^2 = ([d+l]-[d+w])^2$; next, w is found as $\langle l+w+d\rangle-(d+l) = 3$, d as $(d+w)-w$, and l as $(d+l)-d$.

Entities that are found but not named in the text are marked ⟨ ⟩.

[14] With the only difference that the division in (3) is formulated as the question "9 steps of what shall I go in order to have 36", whereas the present problem multiplies by the reciprocal of $d+l$. Since this method is distinctively Babylonian and thus irrelevant for the questions of borrowing and influence I shall not mention it further on.

(14) l+w+d = 70, A = 420; d is found as $\frac{½\cdot([l+w+d]^2-2A)}{l+w+d}$ = 29.

(15) l–w = 7, A = 120; ⟨l+w⟩ is found as $\sqrt{(l-w)^2+4A}$ = 23, w as ½·(⟨l+w⟩–[l–w]) = 8, l as w+(l–w).

(16) A cup weighing 1 mina is composed of gold and copper in ratio 1:9.

(17) l+w+d = 12, A = 12; solved as (14), d = 5.

(18) l+w+d = 60, A = 300; not followed by a solution but by a rule formulated in general terms and corresponding to (14) and (17).

(19) l+d = 45, w+d = 40; again, a general rule is given which follows (13).

(Several problems follow, too damaged however to allow interpretation)

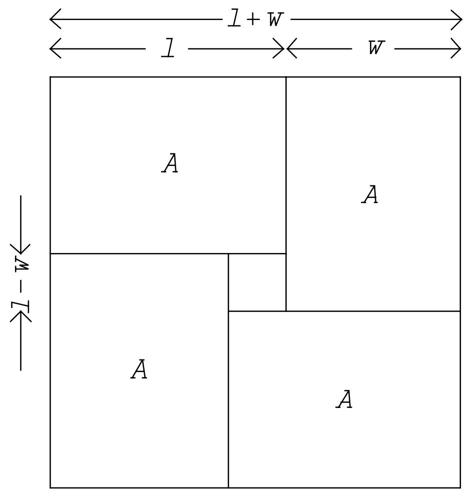

Figure 2. The diagram underlying BM 34568, no. (9) and (15).

Though not occurring as problems of the simple form of (2), (5) and (7), the Pythagorean theorem (or, as I shall prefer to call it in a context where "theorems" have no place, the "Pythagorean rule") is familiar from the Old Babylonian corpus.[15] So is of course the area computation of (6) and (8). No. (2), (5), (6), (7) and (8) thus present us with no innovation beyond the numerical parameters.

The use of a special rule for the case where l and w are in ratio 4:3, on the other hand, is certainly an innovation, even though this ratio was the standard assumption of the Old Babylonian calculators and the reason that 1;15 (= 1¼) appears in tables of technical coefficients as "the coefficient for the diagonal of the length and width".[16] In later times, a standard rule connected the ratios 3:4:5 with constant

[15] [Høyrup 1999b] presents a complete survey of its published appearances.

[16] TMS III, 35, ed. [Bruins & Rutten 1961: 26]. "Length and width" stands for the simplest configuration determined by a single length and a single width, that is, the rectangle.

differences;[17] it is a reasonable assumption that (1) is a first extant witness of this rule.

As problems, (9) and (15) are familiar from Old Babylonian texts, and they are likely to represent the very beginning of the tradition for mixed second-degree riddles – cf. note 7. The solutions, however, are not the traditional ones based on average $\mu = \frac{l+w}{2}$ and deviation $\delta = \frac{l-w}{2}$; instead, they follow the diagram of Figure 2, which is also likely to have been the actual basis for the reasoning. It is a striking stylistic feature (and also a deviation from earlier habits) that even the possibility to determine l and w by a symmetric procedure (namely as $\frac{(l+w)+(l-w)}{2}$ and $\frac{(l+w)-(l-w)}{2}$, respectively) is not used.

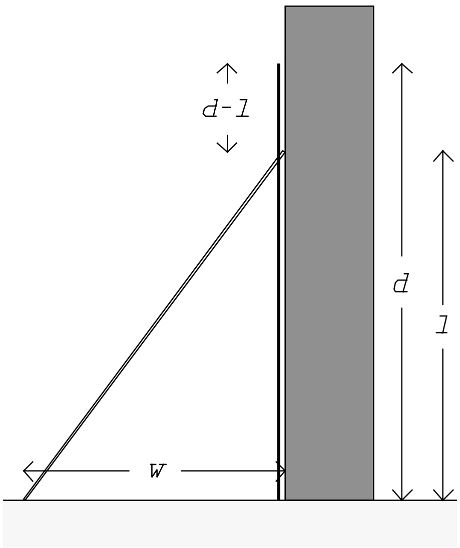

Figure 3. The reed leaned against the wall.

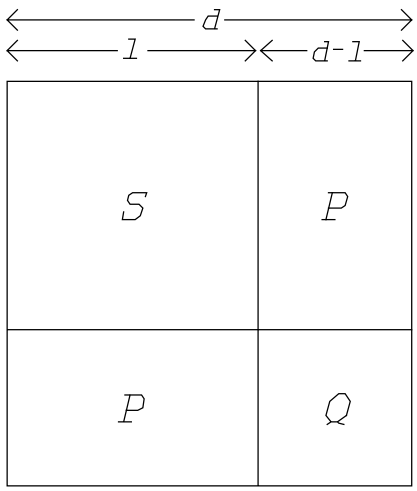

Figure 4. A geometric justification for the solution of BM 34568 no. 12.

Problems treating of a reed or pole leaned against the wall are already present in an Old Babylonian anthology text BM 85196[18]. The situation is shown in Figure 3: A reed of length d first stands vertically against a wall; afterwards, it is moved to a slanted position, in which the top descends to height l (the descent thus being $d–l$) while the foot moves a distance w away from the wall.

In the Old Babylonian versions, d is given together with either w or $d–l$. Solution thus requires nothing beyond simple application of the Pythagorean rule. We cannot know, of course, whether the problem type of the present tablet ($d–l$ and w given) was also dealt with in Old Babylonian times; if so, however, the solution would

17 Referred to in the *Liber mensurationum* [ed. Busard 1968: 97] and in Leonardo's *Pratica* [ed. Boncompagni 1862: 70].

18 Ed. Neugebauer in [MKT II, 44]. On the wide diffusion of this problem type, see [Sesiano 1987].

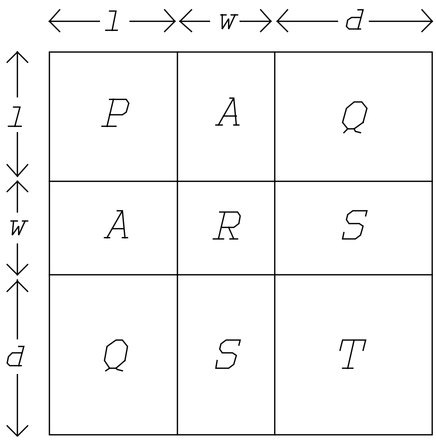

Figure 5. The geometric justification for the case $l+w+d = \alpha$, $A = \beta$.

probably have taken advantage of the facts that $\square(w) = \square(d)-\square(l) =$ ⊏⊐$(2l+2w, \frac{l-w}{2})$ – possibly expressed as ⊏⊐$(l+w,l-w)$, cf. above, p. 3 – and thus have found $d+l$ as $w^2/(d-l)$.[19]

This, as we see, is not done here. In algebraic symbolism, the solution follows from the observation that $(d-l)^2+w^2 = d^2+l^2-2dl+w^2 = 2d^2-2ld = 2d\cdot(d-l)$. It can also be explained from the diagram of Figure 4: $(d-l)^2$ corresponds to the area Q, whereas w^2 corresponds to $\square(d)-\square(l) = \square(d)-S = 2P+Q$. The sum of the two hence equals $2P+2Q = 2$⊏⊐$(d,d-l)$. In itself, this is nothing but a possible basis for the argument, though supported by the fact that halving precedes division by $d-l$, which makes best sense if the doubled rectangle is reduced first to a single rectangle; seen in the light of what follows imminently (and since halving *invariably* precedes division by the measure of a side in parallel cases), this or something very similar seems to have been the actual argument.

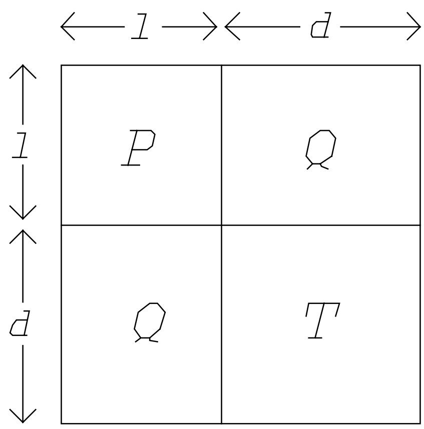

Figure 6. The probable geometrical argument for the type $d+l = \alpha$, $w = \beta$.

Most in favour is obviously the type represented by (14), (17) and (18). The procedure can be explained from Figure 5: $(l+w+d)^2$ is represented by $P+R+T+2A+2Q+2S$; removing $2A$ and making use of the fact that $P+R = T$ we are left with $2Q+2S+2T = 2$⊏⊐$(d,l+w+d)$.

This proof is given by Leonardo Fibonacci in the *Pratica geometrie*, in a way which indicates that he has not invented it himself – at most he has inserted a diagonal (omitted here) in order to explain the construction

[19] Here and in the following, $\square(s)$ designates the (measured or measurable) square with side s, and ⊏⊐(l,w) the rectangle contained by l and w.

of the diagram in Euclidean manner.[20] In view of the faithfulness of both Leonardo and Abū Bakr when they repeat the “new” procedures of the Seleucid style (see below), it is a reasonable assumption that the geometric proofs go back to the same source.

Of the remaining problem types, two are quite new and one only new as far as the procedure is concerned. The type of (3), (4) and (11) can be regarded as a counterpart of (12) (the reed problem). The similarity to this type and to that of (14), (17) and (18) makes it reasonable to look for an analogous justification – see Figure 6: the whole square represents $(l+d)^2$; if we notice that $T = P+w^2$ and remove w^2, we are left with $2P+2Q = 2\sqsubset\!\sqsupset(l,l+d)$.

The type of (13) and (19) is likely to be based on a slight variation of Figure 5, shown here in Figure 7 (which might equally well have served for the type $l+w+d = \alpha$, $A = \beta$, but which happens not to be the proof given by Leonardo). As we see, $(d+l)^2+(d+w)^2 = (P+2Q+R)+(R+2S+T) = (l+w+d)^2+(R–2A)$. That $R–2A$ equals $\square(l–w)$ was familiar knowledge since Old Babylonian times; it follows from Figure 2 if we draw the diagonals of the four rectangles A, as shown in Figure 8 (cf. presently); if we express R as $\square(l)+\square(w)$ it can also be seen from the “naive” version of *Elements* II.7 used in Figure 4, which is likely to have been in still longer use.

The last type is (10). In this exact form it is not found in earlier Babylonian texts,

[20] Ed. [Boncompagni 1862: 68]. Firstly, it is evident from scattered remarks in the work that Leonardo renders what he has found; secondly, all steps of the procedure correspond to what is given in the *Liber mensurationum* [ed. Busard 1968: 97], but in the passage in question there is no trace of verbal agreement with Gherardo’s translation (in other places Leonardo follows Gherardo *verbatim*, correcting only the grammar). Moreover, Leonardo’s statement runs as follows:

> Si maius latus et minus addantur cum dyametro, et sint sicut medietas aree; et area sit 48

whereas Gherardo has

> aggregasti duo latera eius et diametrum ipsius et quod provenit, fuit medietas 48, et area est 48.

Gherardo’s “medietas 48” instead of “24” is obviously meaningless unless it is already presupposed that this 48 represents the area – in other words, that the sum of all four sides and both diagonals equals the area. We may therefore conclude that Leonardo had access to another version of Abū Bakr’s work, in which however the proof was given, or to some closely related work.

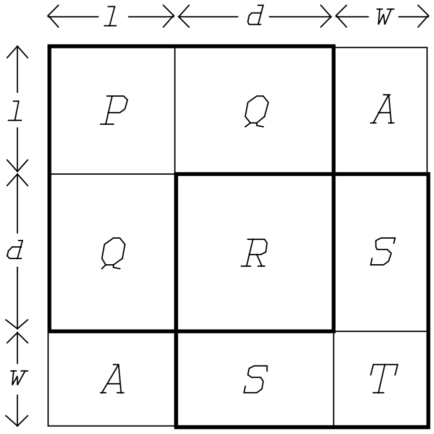

Figure 7. The probable geometric justification for the case $d+l = \alpha$, $d+w = \beta$.

but the closely related problem $d = \alpha$, $A = \beta$ is found in the tablet Db$_2$-146.[21] There it is solved by subtracting $2A$ from □(d), which leaves □(l–w), cf. Figure 8. l and w are then determined from $\frac{l+w}{2}$ and $\frac{l-w}{2}$, that is, average and deviation. This problem recurs with the same procedure in Savasorda's *Collection on Mensuration and Partition* (the *Liber embadorum*)[22]; Abū Bakr and Leonardo[23] use the complementary method and add $2A$ to □(d), finding thus l+w. All three go on with average and deviation.

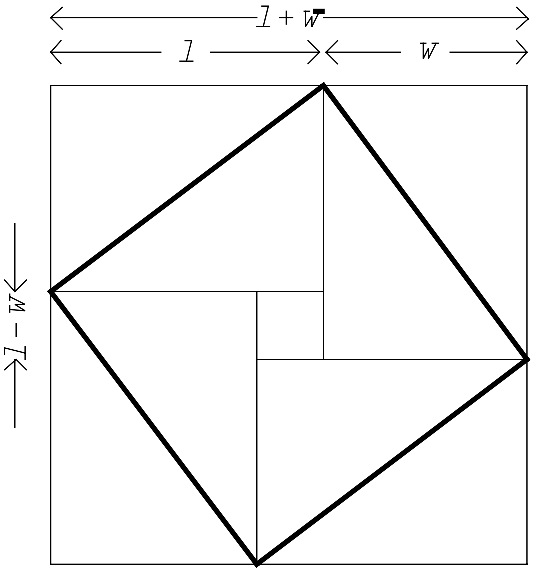

Figure 8. The diagram underlying Db$_2$-146 and BM 34568, no. 10.

Our Seleucid text starts by finding $2A$, namely as □(l+w)–□(d), and next calculates □(l–w) as □(l+w)–$4A$.[24] The rest follows the asymmetric procedure of (9).

Two other cuneiform problem texts of

[21] Ed. [Baqir 1962].

[22] Ed. [Curtze 1902: 48] (Plato of Tivoli's Latin translation); ed., trans. [Guttmann & Millàs i Vallicrosa 1931: 44] (Catalan translation from a somewhat different recension of the Hebrew text).

[23] Ed. [Busard 1968: 92] and [Boncompagni 1862: 64], respectively.

[24] This seemingly roundabout procedure is the clearest evidence that the configuration of Figure 8 is indeed used. From a strictly algebraic point of view, it would be simpler to find $(l–w)^2$ as $2d^2–(l+w)^2$.

Seleucid date are known. VAT 7848[25] contains geometric calculations of no interest in the present context. AO 6484 was already mentioned. It is a mixed anthology, which is relevant on three accounts:

1. One of its problems (obv. 12, statement only) is a rectangle problem of the type $l+w+d = \alpha$, $A = \beta$.
2. It is interested in the summation of series "from 1 to 10". In obv. 1–2, $1+2+...+2^9$ is found, in obv. 3–4 $1+4+...+10^2$ is determined. The latter follows the formula

$$Q_n = \sum_{i=1}^{n} i^2 = (1 \cdot \frac{1}{3} + n \cdot \frac{2}{3}) \cdot \sum_{i=1}^{n} i \ .$$

3. It contains a sequence of *igûm-igibûm* problems (see above, p. 4), and thus demonstrates that the tradition of second-degree algebra had not been totally interrupted within the environment that made use of the sexagesimal place value system with appurtenant tables of reciprocals. They differ from the Old Babylonian specimens by dealing with numbers containing up to four significant sexagesimal places – no doubt an innovation due to the environment of astronomer-priests where it was produced,[26] and in which multi-place computation was routine.

So far there is no particular reason to believe that the other innovations of which the Seleucid texts are evidence were also due to this environment – nor *not* to believe it. Although BM 34568 is theoretically more coherent than AO 6484 it is still a secondary mixture – as shown by the presence of the alligation problem, which is certainly no invention of the astronomers.

Demotic evidence

At this point, two Demotic papyri turn out to be informative: P. Cairo *J.E.89127–30,89137–43* and P. British Museum *10520*,[27] the first from the third century BCE, the second probably of early(?) Roman date.

The latter begins by stating that "1 is filled up twice to 10", that is, by asking for the sums $T_{10} = \sum_{i=1}^{10} i$ and $P_{10} = \sum_{i=1}^{10} T_i$ and answering from the correct formulae

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[25] Ed. Neugebauer & Sachs in [MCT, 141].

[26] The colophon of the tablet tells that it was produced by Anu-aba-utēr, who identifies himself as "priest of [the astrological series] *Inūma Anu Enlil*", and who is known as a possessor and producer of astronomical tablets.

[27] Both ed., trans. [Parker 1972].

$$T_n = \frac{n^2+n}{2}\ ,\ \ P_n = (\frac{n+2}{3})\cdot(\frac{n^2+n}{2})$$

– not overlapping with the series dealt with in AO 6484 but sufficiently close in style to be reckoned as members of a single cluster.[28]

The Cairo Papyrus is more substantial for our purpose. Firstly it contains no less than 7 problems about a pole first standing vertically and then leaned obliquely against a wall (cf. above, p. 8 and Figure 3). Three are of the easy type where *d* and *w* given, and solved by simple application of the Pythagorean rule; three are of the equally simple type where *d* and *d–l* are given, and solved similarly (both, we remember, are treated in an Old Babylonian text). Two, finally, are of the more intricate type found in BM 34568 (*w* and *d–l* given) and solved as there.

Further on in the same papyrus, two problems about a rectangle with known diagonal and area are found. The solution is clearly related to the problems of BM 34568 (cf. also Figure 8): Addition of 2*A* to ☐(*d*) and subsequent taking of the square root gives *l*+*w*, whereas subtraction and taking of the root yields *l*–*w*. *l* and *w* are then found by the habitual asymmetric procedure.

These and other Demotic mathematical papyri also contain material that descends from the Pharaonic mathematical tradition as we know it from the Rhind and Moscow Papyri. What they share with the Seleucid tablets, however, has no known Egyptian antecedents (neither in actual content nor in style); in some way it represents an import (there is no reason to doubt the West Asian origin of the reed

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28 All our evidence for this cluster is second-century BCE or later. Yet, since no other traces of Greek influence is found in these texts, the interest in square, triangular and pyramid numbers and in the sacred 10 of the Pythagoreans is striking.

Even more striking is the fact that the Demotic determination of T_{10} as $\frac{10^2+10}{2}$ corresponds to an observation made by Iamblichos in his commentary to Nicomachos's *Eisagoge*, *viz* that 1+2+...+9+10+9+...+2+1 = 10×10 [Heath 1921: I, 114], whereas the Seleucid determination of $1^2+2^2+...+10^2$ as $(1\cdot\frac{1}{3}+10\cdot\frac{2}{3})\cdot\sum_{i=1}^{10} i$ turns up in the pseudo-Nichomachean *Theologoumena arithmeticae* (X.64, ed. [de Falco 1975: 86], trans. [Waterfield 1988: 115]), in a quotation from Anatolios.

More precisely, Anatolios gives the sum as "sevenfold" $\sum_{i=1}^{10} i$, that is, in a form from which the correct Seleucid formula cannot be derived – another hint that the Pythagorean knowledge of the formula was derivative.

problem, nor the ultimate descent of the rectangle problems from surveyors' tradition). It is clear, however, that they represent the new stage at least as well as the Seleucid texts. This does not prove that Demotic Egypt was the place where the transformation occurred; but the appearance of the characteristic problems as fully integrated components of Egyptian scribal mathematics in the third century BCE makes it implausible that the novelty be due to the environment of Seleucid astronomer-priests. Indeed, West Asian taxators and surveyors will certainly have followed the Assyrian and Achaemenid conquerors to Egypt; on the other hand, the methods of Babylonian mathematical astronomy, when eventually reaching Egypt, did so in a reduced version which shows them to have been carried by astrological "low" practitioners, not by the scholar-astronomers that created them.

Abū Bakr and Leonardo

The evidence offered by the *Liber mensurationum* and Leonardo's *Pratica* seems to speak more generally against a localization of the innovations within the surveyors' core tradition (however this core looked in the late first millennium BCE – but the chain that transmitted not only problems but also standard phraseology can reasonable be regarded as a "core"). With slight variations, Abū Bakr has all, and Leonardo almost all of the problems from BM 34568.[29] However, all problems that originated in earlier epochs (given $l+w$ and $l–w$, given A and $l±w$, given A and d, given $l+w$ and d) are solved traditionally, by means of average and deviation; asymmetric procedures occur only in connection with the definitely *new* problems. The general shift to asymmetry which we encounter in BM 3456 and the Demotic papyri was evidently not accepted in the core tradition, although it accepted the new problems and did not attempt to reformulate these in more symmetric ways.[30]

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[29] The "reed problem" occurs in regular rectangle version, the problem $d+l = \alpha$, $d+w = \beta$ as $d+w = \beta$, $l = w+\gamma$ (which explains the error in the formulation of BM 34568, where $(l–w)^2$ appears as an unexplained 1).

[30] In another respect, however, even the new problem types as appearing with Abū Bakr and Leonardo give evidence of normalization, cf. above, note 20: the problem $l+w+d = 24$, $A = 48$ is formulated in a way which shows it beyond reasonable doubt to have been derived from a problem ${}_2l+{}_2w+{}_2d = A = 48$ (subscript "${}_2$" meaning "both"). This predilection for "the perimeter" in rectangle problems is related to what circulated in Neopythagorean environments and to what we find with Mahāvīra; in contrast, all Babylonian sources, Old Babylonian as well as Seleucid, are interested in "both sides" (cf. also BM 34568 no. 17, $l+w+d = A = 12$)

This would agree badly with emergence of the new problems within the core.

Greco-Roman problems

Relevant material from Greek and Latin sources is extremely scarce. Only two texts that I know of contain evidence of influence from the new "Seleucid" style.

One is the Latin *Liber podismi*,[31] the title of which betrays it to be a translation from a Greek original (or at least to be inspired by a Greek model). One of its problems deals with the rectangle (actually a right triangle, as preferred in all Greek sources) with given area and diagonal, and does so in the manner of the Demotic papyrus, with the small difference that l is found first as $\frac{1}{2}\cdot(\langle l+w\rangle+\langle l-w\rangle)$, and w next as $l-\langle l-w\rangle$. The next problem is overdetermined, giving A, d and $l+w$; it first finds $\langle l-w\rangle$ from d^2 and A, next l as $\frac{1}{2}\cdot([l+w]+\langle l-w\rangle)$, and finally w as $(l+w)-l$. Obviously, the solutions of BM 34568 no. 10 and the Demotic diagonal-area problems are combined, but with the same variant of the asymmetric method as in the previous problem.

The other is Papyrus graecus genevensis 259[32]. It contains three problems on right triangles:

1. $w = 3$, $d = 5$
2. $w+d = 8$, $l = 4$
3. $l+w = 17$, $d = 13$

The first tells us nothing. The second (identical with BM 34568 no. 4) makes use of the fact that $l^2 = d^2-w^2 = (d+w)\cdot(d-w)$[33] and finds $\langle d-w\rangle$ as $l^2/(d+w)$; w is then found as $\frac{1}{2}\cdot([d+w]+\langle d-w\rangle)$, and finally d as $(d+w)-w$. The third – identical with BM 34568 no. 10 apart from the numerical parameters – is solved in an algebraically straighter way (cf. note 24): $\langle(l-w)^2\rangle$ is found as $2d^2-(d+w)^2$, w as $\frac{1}{2}\cdot([l+w]-\langle l-w\rangle)$, and l finally as $(l+w)-w$.

All in all, these Greek problems might belong in the periphery of a cluster where

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31 Ed. [Bubnov 1899: 511*f*]. The two relevant problems are reproduced in [Sesiano 1998: 298*f*].

32 Ed. [Sesiano 1999].

33 The trick, we notice, which was *not* used in the Seleucid reed-problem, cf. p. 9. On the other hand, it appears to be used in the "pre-Euclidean" version of the computation of the height of a scalene triangle (see [Høyrup 1997a: 82]), and in problems about two concentrically located squares; it will thus have been quite familiar (cf. also the version presented by *Elements* II.8).

the Demotic and Seleucid texts represent something closer to the core: they deviate slightly in their choice of actual procedures, but the general tenor is the same. They share the asymmetric approach, but may have replaced the geometric visualization by more genuine algebraic manipulation. Further, they are formulated in terms of right triangles instead of rectangles.

An Indian witness: Mahāvīra

Mahāvīra's 9th century *Gaṇita-sāra-saṅgraha* is in itself a late source – slightly later than al-Khwārizmī and roughly contemporary with Thābit ibn Qurrah.[34] There are, however, no reasons to doubt Mahāvīra's assertion that he has taken advantage of "the help of the accomplished holy sages" when "glean[ing] from the great ocean of the knowledge of numbers a little of its essence [...] and giv[ing] out [...] the *Sārasaṅgraha*, a small work on arithmetic",[35] that is, that he presents what was since times immemorial part of the Jaina tradition; moreover, internal evidence also speaks in favour of the claim.[36] Given the history of the Jaina community, the adoption of material from the Near East or the Mediterranean region of which Mahāvīra's work bears witness is likely to have taken place in late pre-Christian or at most early Christian centuries.

As a matter of fact, we should probably rather speak of "adoptions" in the plural. Mahāvīra divides his chapter on geometry into four sections: "approximate measurement"; "minute accurate calculation"; "devilishly difficult problems"; and one on the "Janya operation", which does not concern the present argument. All groups encompass material that is not known from non-Indian sources, but all also contain rules that are familiar from the Near East and the Mediterranean.

In the section on "approximate" area measurement (pp. 187–197, stanzas VII.7–48) we find the "surveyors' formula"; the determination of the area of a circular ring as average circumference times width (indeed exact); the rule that the circular circumference is three times the diameter; and the problem of finding the separate value of circumference, diameter and area from their sum (in this order, the characteristic order of the pre-Old-Babylonian riddle tradition) together with a corresponding rule (based on the choice of the circumference as the basic parameter and on $\pi = 3$).

The section on "minutely accurate" area determination (pp. 197–208, stanzas

[34] I am grateful to Yvonne Dold-Samplonius for having first directed my attention to this important source.

[35] Ed., trans. [Raṅgācārya 1912: 3]. All subsequent page references are to this translation.

[36] See the discussion in [Høyrup 1998: 53*f*], which I shall not repeat here.

VII.49–89½) gives the "pre-Euclidean" method for finding the (inner) height in a scalene triangle, together with Hero's formula for triangular and quadrangular areas (for the latter set forth as if it were of general validity); gives the circular area as one fourth of the product of arc and diameter;[37] and states the circumference to be √10 times the diameter.

Among the "devilishly difficult" problems (pp. 220–257, stanzas 112½–232½) we find several of the characteristic traditional surveyors' riddles (together with variants with "non-natural" coefficients corresponding to transformation of the tradition within an institutionalized school environment): area equal to perimeter (for squares and rectangles);[38] and later on, rectangular area and perimeter given (a slight variation of the problem $A = \alpha$, $l+w = \beta$); rectangular perimeter and diagonal given (equivalent to problem 10 of BM 34568 and problem 3 of the Geneva Papyrus); and rectangular area and diagonal given. The perimeter-and-diagonal problem has the same parameters as the Geneva version and, more strikingly, finds $(l–w)^2$ in the same way, *viz* as $2d^2–(l+w)^2$; the area-and-diagonal problem finds both $l+w$ and $l–w$, as do the Cairo Papyrus and the *Liber podismi*. Even the area-and-perimeter problem goes via $l+w$ and $l–w$, not via average and deviation. But in contrast to all the western versions, l and w are then found by a symmetric procedure with a technical name of its own (*saṅkramaṇa*), as average and deviation of $l+w$ and $l–w$. As in the Seleucid and Demotic problems, rectangles and not right triangles are concerned.

The riddles have thus been adopted in "Seleucid-Demotic", not "Old Babylonian" version; in some respects, however, they echo the Greek rather than the Near Eastern form. The "minutely accurate" calculations are related to developments that will only have taken place in the Near East around the mid-first millennium BCE.[39]

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37 This formula is already used in Old Babylonian material, but only for the semicircle where both arc and diameter are external measures.

38 Of these, only the rectangular variant $A = l+w$ is found in the Old Babylonian material. However, they certainly circulated in the Mediterranean region during the classical epoch: the *Theologoumena arithmeticae* mentions repeatedly that the square □(4) is the only square that has its area equal to the perimeter (II.10, IV.23 [ed. de Falco 1975: 11, 29], trans. [Waterfield 1988: 44, 63]); the second passage cites Anatolios), doing so in a way that demonstrates this to be a traditional observation; Plutarch on his part refers to Pythagorean knowledge of the equality of area and perimeter in the rectangle ⊏⊐(3,6) (*De Iside et Osiride* 42, ed., trans. [Froidefond 1988: 214*f*]).

39 The determination of the height of the scalene triangle must predate Euclid, since

Whatever was borrowed for this section will thus have arrived when the Jaina school was already established; but its arrival may well have preceded that of the "devilishly difficult" problems. Those of the "approximate" rules that were adopted from elsewhere, on the other hand, may have arrived long before the appearance of Jainism.

Chapter VI of Mahāvīra's work contains a section on the summation of series (pp. 168–176, stanzas 290–317) which at first looks similar to the Seleucid-Demotic cluster: arithmetical series, geometrical series, and series of the form $\sum_i N_i$, where N_i are squares, cubes or triangular numbers and i runs through an arithmetical series (with any starting point, any number of members and any difference). What Mahāvīra offers is, however, so much more elaborate than what we find in the Near Eastern texts, and its formulas so different, that inspiration one way or the other becomes a gratuitous hypothesis.

Other Indian sources confirm that such series were a much more serious concern in India at least after c. 500 CE than they seem to have been in the Seleucid-Demotic area; indeed, both Brahmagupta and Bhaskara II deal with the same types as Mahāvīra, and both know[40] that

$$P_n = \frac{n+2}{3} \cdot T_n \quad , \quad Q_n = \frac{1+2n}{3} \cdot T_n \quad .$$

Āryabhaṭa I followed by Bhāskara I[41] give slightly different variants, which however (given the testimony of Brahmagupta and Bhaskara II) are likely to be reformulations of the same formulae. All, however, determine T_n from the general

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Elements II.13 can reformulate it and II.12 extend it to the case of the outer height; but the absence from the pre-Seleucid but still Late Babylonian tablets published in [Friberg, Hunger & al-Rawi 1990] and [Friberg 1997] shows that the invention cannot predate 500 BCE by much, if at all.

40 Trans. [Colebrooke 1817: 290–294] and [Colebrooke 1817: 51–57], respectively. Both also know that $C_n = \sum_{i=1}^{n} i^3 = T_n{}^2$, a formula that also turns up in al-Karajī's *Fakhrī* [Woepcke 1853: 61], in a context which suggests this formula to have belonged to the same cluster.

41 *Āryabhaṭīya* II.21–22, ed., trans. [Clark 1930: 37]; I thank Agathe Keller for giving me access to her as yet published work on Bhāskara's commentary [Keller 2000].

formula for the sum of an arithmetical series, $S = [\frac{(n-1)d}{2} + a]\cdot n$, n being the number of terms, a the first term, and d the difference – thus as the average term multiplied by the number of terms, as they indeed explain. This formula is also used in the Bakhshālī manuscript (likely to be somewhat earlier than Brahmagupta),[42] from which the more complex sums (P_n, Q_n, C_n) are all absent. The formula for S is obviously based on a purely arithmetical consideration, whereas the Demotic-Seleucid-Pythagorean formula appears to be derived from considerations based on *psephoi* (cf. note 28). All in all, independent development in the two areas followed by cross-fertilizations appears to be the most plausible explanation.

Nine Chapters on Arithmetic

The Chinese *Nine Chapters on Arithmetic*[43] are roughly contemporary with our Seleucid and Demotic sources. Though in the main a witness of an independent tradition they do contain problems that appear to point to the Near East and the Mediterranean – not least a version of the "reed against the wall" in IX.8.

On a first inspection, the whole sequence IX.6–13 looks as if it were related to the Seleucid and Demotic rectangle problems, even though formulated in varying dress and mostly so as to deal with right triangles (the right triangle is indeed the topic of the chapter as a whole). On closer inspection, however, the evidence turns out to be inconclusive.

Translated into the usual l,w,d symbolism – that is, expressed in the way which will make kinship stand out as clearly as possible – the problems and their solutions are the following:

(6) $d–l = 1$, $w = 5$ (the analogue of the Seleucid reed problem BM 34568 no. 12). l is found as $\frac{w^2-(d-l)^2}{2\cdot(d-l)}$, whence d, where BM 34568 finds d as $\frac{½\cdot(w^2+[d-l]^2)}{d-l}$ and next l. We observe that the Chinese procedure does not halve before dividing by ($d–l$), which suggests that a geometric justification, *if* once present, had been forgotten.

(7) $d–l = 3$, $w = 8$. The structure of the problem is the same, but the solution proceeds differently: $\langle d+l\rangle$ is found as $\frac{w^2}{d-l} = \frac{\langle d+l\rangle\cdot(d-l)}{d-l}$, and d as $½\cdot(\langle d+l\rangle+[d–l])$. This is related to the second problem of the Geneva papyrus,

42 Ed.,trans. [Hayashi 1995: 439 and *passim*]

43 Ed., trans. [Vogel 1968].

but different from everything in the Seleucid and Demotic texts (cf. also note 33).

(8) $d–l = 1$, $w = 10$. Same problem type and same procedure as no. 7.

(9) Another variation of no. 7.

(10) Yet another variation of no. 7.

(11) $d = 100$, $l–w = 68$. $\langle\frac{l+w}{2}\rangle$ is found as $\sqrt{\frac{d^2-2\cdot(\frac{l-w}{2})^2}{2}}$, and w then as $½\cdot(\langle\frac{l+w}{2}\rangle+\langle\frac{l-w}{2}\rangle)$ – certainly not Seleucid-Demotic in style with its use of average and deviation, nor however similar in detail to anything from the older Near Eastern tradition.

(12) $d–l = 2$, $d–w = 4$. The solution builds on the observation that $\sqsubset\!\sqsupset(d–[d–l]–[d–w]) = 2(d–l)\cdot(d–w)$. This can be argued from a diagram similar in style to Figure 7, which served for the problem type $d+l = \alpha$, $d+w = \beta$ – namely the one shown in Figure 9: the full square $\square(d)$ must equal the sum of the squares $\square(l)$ and $\square(w)$; therefore, the overlap $S = \square(d–[d–l]–[d–w])$ must equal the area which they do not cover, that is, $2R = 2\sqsubset\!\sqsupset(d–l,d–w)$.[44]

(13) $d+l = 10$, $w = 3$. Solved as no. 2 of the Geneva papyrus: $[d–l]$ is found as $w^2/(d–l)$, and l as $½\cdot([d+l]+[d–l])$.

Similarities are certainly present, and the appearance of the pole leaned against the wall seems to suggest that contact has played a role. But the similarities are always relative; *if* the whole interest in this problem type and the approach used in the solution is ultimately inspired from abroad (which does *not* follow from a plausible borrowing of the pole-against-wall dress), then the use of average and deviation in no. 11 and the predominant use of the identity $w^2 = (d+l)\cdot(d–l)$ would rather point

[44] It should be observed that we are in the case of constant differences, $d–l = l–w$. The Chinese solution, of course, does not take advantage of that, and we might therefore claim that the closest kin in the western sources is the case of arbitrary differences. This case is not treated by Abū Bakr (nor in the ancient texts); it *is* treated by Leonardo [ed. Boncompagni 1862: 71], but he solves it by means of algebra, not by a rule or a diagram pointing toward the tradition. Even though the Chinese solution may build on a geometric argument similar to the ones known from Leonardo (etc.), nothing allows us to connect the actual problem or procedure to Near Eastern or Mediterranean texts.

to a contact preceding the Seleucid epoch and ensuing independent development – but even that is certainly no necessary conclusion.

It is highly risky to base the construction of a stemma on the transformations undergone by a single phrase, in particular when crosswise contamination is possible. *If* we are none the less tempted to engage in a similar perilous game we may notice that those of our sources that make preferential use of the identity $w^2 = (d+l)\cdot(d-l)$ are those that formulate problems in terms of right triangles, not quadrangles; if not allowing any positive conclusions the observation may at least remind us that the "Seleucid" innovations need not have emerged together, however much the scarcity of sources tempts us to see them as belonging together.

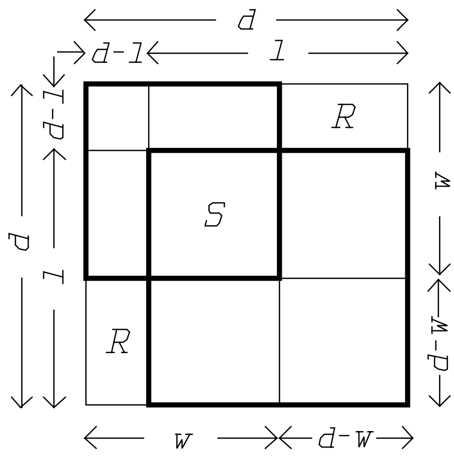

Figure 9. The possible geometric basis for problem IX.12 of the *Nine Chapters*.

This observation may be generalized into a conclusion: the written sources that reflect diffusion of the particular "Seleucid" problem types outside the Near Eastern area are, like the sources *from* the Near East itself, too few and too diverse to allow any certain conclusions concerning the details of the transmission pattern.

Bibliography

Baqir, Taha, 1962. "Tell Dhiba'i: New Mathematical Texts". *Sumer* **18**, 11–14, pl. 1–3.

Boncompagni, Baldassare (ed.), 1862. *Scritti* di Leonardo Pisano matematico del secolo decimoterzo. II. *Practica geometriae* et *Opusculi*. Roma: Tipografia delle Scienze Matematiche e Fisiche.

Bruins, E. M., & M. Rutten, 1961. *Textes mathématiques de Suse*. (Mémoires de la Mission Archéologique en Iran, XXXIV). Paris: Paul Geuthner.

Bubnov, Nicolaus (ed.), 1899. Gerberti postea Silvestri II papae *Opera mathematica* (972—1003). Berlin: Friedländer.

Busard, H. L. L., 1968. "L'algèbre au moyen âge: Le «Liber mensurationum» d'Abû Bekr". *Journal des Savants*, Avril-Juin 1968, 65–125.

Clark, Walter Eugene (ed., trans.), 1930. The *Āryabhaṭīya* of Āryabhata. Chicago: University of Chicago Press.

Colebrooke, H. T. (ed., trans.), 1817. *Algebra, with Arithmetic and Mensuration from the Sanscrit of Brahmagupta and Bhascara*. London: John Murray.

Curtze, Maximilian (ed.), 1902. *Urkunden zur Geschichte der Mathematik im Mittelalter und der Renaissance*. (Abhandlungen zur Geschichte der mathematischen Wissenschaften, vol. 12–13). Leipzig: Teubner.

de Falco, V. (ed.), U. Klein (add., corr.), 1975. [Iamblichi] *Theologumena arithmeticae*. [2]Stuttgart: Teubner ([1] 1922).

Farber, Walter, 1993. "Zur Orthographie von *EAE* 22: Neue Lesungen und Versuch einer Deutung", pp. 247–257 in Hannes D. Galter (ed.), *Die Rolle der Astronomie in den Kulturen Mesopotamiens*. Beiträge zum 3. Grazer Morgenländischen Symposion (23.–27. September 1991). (Grazer Morgenländische Studien, 3). Graz.

Friberg, Jöran, 1997. "'Seed and Reeds Continued'. Another Metro-Mathematical Topic Text from Late Babylonian Uruk". *Baghdader Mitteilungen* **28**, 251–365, pl. 45–46.

Friberg, Jöran, Hermann Hunger & Farouk al-Rawi, 1990. "»Seeds and Reeds«: A Metro-Mathematical Topic Text from Late Babylonian Uruk". *Baghdader Mitteilungen* **21**, 483–557, Tafel 46–48.

Froidefond, Christian (ed., trans.), 1988. Plutarque, *Isis et Osiris*. (Oeuvres morales V.2). Paris: Les Belles Lettres.

Guttman, Miquel, & J. Millas y Vallicrosa (eds, trans.), 1931. Abraam bar Hiia. *Llibre de geometria. Hibbur hameixihà uebatixbòret*. (Biblioteca Hebraico-Catalana, 3). Barcelona: Alpha.

Hayashi, Takao, 1995. *The Bakhshālī Manuscript: An Ancient Indian Mathematical Treatise*. (Groningen Oriental Studies, 11). Groningen: Egbert Forsten.

Heath, Thomas L., 1921. *A History of Greek Mathematics*. 2 vols. Oxford: The Clarendon Press, 1921.

Høyrup, Jens, 1986. "Al-Khwârizmî, Ibn Turk, and the Liber Mensurationum: on the Origins of Islamic Algebra". *Erdem* (Ankara) **2**, 445–484.

Høyrup, Jens, 1990a. "Sub-Scientific Mathematics. Observations on a Pre-Modern Phenomenon". *History of Science* **28**, 63–86.

Høyrup, Jens, 1990b. "Algebra and Naive Geometry. An Investigation of Some Basic Aspects of Old Babylonian Mathematical Thought". *Altorientalische Forschungen* **17**, 27–69, 262–354.

Høyrup, Jens, 1997a. "Hero, Ps.-Hero, and Near Eastern Practical Geometry. An Investigation of *Metrica*, *Geometrica*, and other Treatises", pp. 67–93 *in* Klaus Döring, Bernhard Herzhoff & Georg Wöhrle (eds), *Antike Naturwissenschaft und ihre Rezeption*, Band 7. Trier: Wissenschaftlicher Verlag Trier. (For obscure reasons, the publisher has changed □ into ~ and ▭ into ¤§ on p. 83 after having supplied correct proof sheets).

Høyrup, Jens, 1997b. "Lasciti sotto-scientifici alla matematica d'abbaco: quasi-algebra ed altre strane specie". *Filosofi og Videnskabsteori på Roskilde Universitetscenter*. 3. Række: *Preprints og Reprints* 1997 Nr. 2.

Høyrup, Jens, 1997c. "Mathematics, Practical and Recreational", pp. 660–663 *in* Helaine Selin (ed.), *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*. Dordrecht etc.: Kluwers.

Høyrup, Jens, 1997d. "Algebra, surveyors'", pp. 52–55 *in* Helaine Selin (ed.), *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*. Dordrecht etc.: Kluwers.

Høyrup, Jens, 1998. "On a Collection of Geometrical Riddles and Their Role in the Shaping of Four to Six 'Algebras'". *Filosofi og Videnskabsteori på Roskilde Universitetscenter*. 3. Række: *Preprints og Reprints* 1998 Nr. 2. To be published in *Science in Context* (2001).

Høyrup, Jens, 1999a. "The Finer Structure of the Old Babylonian Mathematical Corpus. Elements of Classification, with some Results", pp. 117–177 *in* Joachim Marzahn & Hans Neumann (eds), *Assyriologica et Semitica*. Festschrift für Joachim Oelsner. (Altes Orient und Altes Testament, Band 252). Kevelaer: Butzon & Bercker / Neukirchen-Vluyn: Neukirchener Verlag. In print.

Høyrup, Jens, 1999b. "Pythagorean 'Rule' and 'Theorem' – Mirror of the Relation Between Babylonian and Greek Mathematics", pp. 393–407 *in* Johannes Renger (ed.), *Babylon: Focus mesopotamischer Geschichte, Wiege früher Gelehrsamkeit, Mythos in der Moderne*. 2. Internationales Colloquium der Deutschen Orient-Gesellschaft, 24.–26. März 1998 in Berlin. Berlin: Deutsche Orient-Gesellschaft / Saarbrücken: SDV Saarbrücker Druckerei und Verlag.

Keller, Agathe, 2000. "Un commentaire indien du VIIème siècle. Bhāskara et le *gaṇita-pāda* de *Āryabhatīya*". *Thèse de Doctorat*, Université Paris VII, 2000.

MCT: O. Neugebauer & A. Sachs, *Mathematical Cuneiform Texts*. (American Oriental Series, vol. 29). New Haven, Connecticut: American Oriental Society, 1945.

MKT: O. Neugebauer, *Mathematische Keilschrift-Texte*. I-III. (Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik. Abteilung A: Quellen. 3. Band, erster-dritter Teil). Berlin: Julius Springer, 1935, 1935, 1937.

Parker, Richard A., 1972. *Demotic Mathematical Papyri*. Providence & London: Brown University Press.

Raṅgācārya, M. (ed., trans.), 1912. The *Gaṇita-sāra-sangraha* of Mahāvīrācārya with English Translation and Notes. Madras: Government Press.

Rebstock, Ulrich, 1993. *Die Reichtümer der Rechner (Ġunyat al-Ḥussāb) von Aḥmad b. Ṯabāt (gest. 631/1234). Die Araber – Vorläufer der Rechenkunst*. (Beiträge zur Sprach- und Kulturgeschichte des Orients, 32). Walldorf-Hessen: Verlag für Orientkunde Dr. H. Vorndran.

Saliba, George A., 1972. "The Meaning of al-jabr wa'l-muqābalah". *Centaurus* **17**, 189–204.

Sesiano, Jacques, 1987. "Survivance médiévale en Hispanie d'un problème né en Mésopotamie". *Centaurus* **30**, 18–61.

Sesiano, Jacques, 1998. "An Early Form of Greek Algebra". *Centaurus* **40**, 276–302.

Sesiano, Jacques, 1999. "Sur le Papyrus graecus genevensis 259". *Museum Helveticum* **56**, 26–32.

Vogel, Kurt (ed., trans.), 1968. *Chiu chang suan shu. Neun Bücher arithmetischer Technik. Ein chinesisches Rechenbuch für den praktischen Gebrauch aus der frühen Hanzeit (202 v. Chr. bis 9 n. Chr.)*. (Ostwalds Klassiker der Exakten Wissenschaften. Neue Folge, Band 4). Braunschweig: Friedrich Vieweg & Sohn.

Waterfield, R. (trans.), 1988. Iamblichos, *The Theology of Arithmetic*. Grand Rapids, Michigan: Phanes.

Woepcke, Franz, 1853. *Extrait du Fakhrî, traité d'algèbre* par Aboû Bekr Mohammed ben Alhaçan Alkarkhî; *précédé d'un mémoire sur l'algèbre indéterminé chez les Arabes*. Paris: L'Imprimerie Impériale.