ALGEBRA AND NAIVE GEOMETRY
An Investigation of Some Basic Aspects of Old Babylonian Mathematical Thought
By Jens Høyrup
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Til Sara og Janne
ABSTRACT

Through a broad structural analysis and a close reading of Old Babylonian mathematical *procedure texts* dealing mainly with problems of the second degree it is shown that Old Babylonian *algebra* was neither a *rhetorical algebra* dealing with numbers and arithmetical relations between numbers nor built on a set of fixed algorithmic procedures. Instead, the texts must be read as *naive* prescriptions for geometric analysis -- naive in the sense that the results are seen by immediate intuition to be correct, but the question of correctness never raised -- dealing with measured or measurable but unknown line segments, and making use of a set of operations and techniques different in structure from that of arithmetical algebra.

The investigation involves a thorough discussion and reinterpretation of the technical terminology of Old Babylonian mathematics, elucidates many terms and procedures which have up to now been enigmatic, and makes many features stand out which had not been noticed before.

The second-last chapter discusses the metamathematical problem, to which extent we are then entitled to speak of an Old Babylonian *algebra*; and the over-all implications of the investigation for the understanding of Old Babylonian patterns of thought. It is argued that these are not *mythopoetic* in the sense of H. and H. A. Frankfort, nor *savage* or *cold* in a Levi-Straussian sense, nor however as abstract and modern as current interpretations of the mathematical texts would have them to be.

The last chapter investigates briefly the further development of Babylonian *algebra* through the Seleucid era, demonstrating a clear arithmetization of the patterns of mathematical thought; the possible role of Babylonian geometrical analysis as inspiration for early Greek geometry; and the legacy of Babylonian *algebraic* thought to Medieval Islamic algebra.
INTRODUCTION

The following contains an account of a broad investigation of the terminology, methods, and patterns of thought of Old Babylonian so-called algebra. I have been engaged in this investigation for some years, and circulated a preliminary and fairly unreadable account in 1984 (of which the item [Hejrup 1985] in the bibliography of the present paper is a slightly corrected reprint). I have also presented the progress of the project in the three Workshops on Concept Development in Babylonian Mathematics held at the Seminar für Vorderasiatische Altertumskunde der Freien Universität Berlin in 1983, 1984 and 1985, and included summaries of some of my results (without the detailed arguments) in various contexts where they were relevant.

This paper is meant to cover my results coherently and to give the details of the argument, without renouncing completely on readability. The present preprint version is circulated in the hope that corrections and critical comments will be sent to the author soon enough to be taken into account in the final journal publication (provided any journal is willing to publish a paper of this length).

The paper contains many discussions of philological details which will hardly be understandable to historians of mathematics without special assyriological training, but which were necessary if the philological specialists should be able to evaluate my results; I hope the non-specialist will not be too disturbed by these stumbling-stones. On the other hand, many points which are trivial to the assyriologist are included in order to make it clear to the non-specialist why current interpretations and translations are only reliable up to a certain point, and why the complex discussions of terminological structure and philological details are at all necessary. I apologize to whoever will find them boring and superfluous.

It is a most pleasant duty to express already at this intermediate stage my gratitude to all those who have assisted me over the years, especially Dr. Bendt Alster, Dr. Aage Westenholz and Dr. Mogens Trolle Larsen of Copenhagen University, and to Professor, Dr. Hans Nissen, Professor, Dr. Johannes Renger, Robert Englund, and Dr. Killian Butz of Freie Universität Berlin, together with all participants in the Berlin Workshops, not least the indefatigable Professor Jöran Friberg of Göteborg.
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Everybody who followed the Berlin Workshops will know that Dr. Peter Damerow of the Max-Planck-Institut für Bildungsforschung, Berlin, deserves the greatest gratitude of all, to which I can add my personal experience as made over the last five years.

The intelligent reader will easily guess who remains responsible for all errors.

I dedicate the work to my daughters Sara and Janne, for reasons which have nothing to do with mathematics, Babylonia or Assyriology, but much with our common history over the years.

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Jens Høyrup
Institute of Educational Research, Media Studies and Theory of Science
Roskilde University Centre
P.O. Box 280
DK-4000 Roskilde
Denmark
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1. The starting point: Numbers or lines -- in method and in conceptualization.

For about 50 years it has been known that the Babylonians of the Old Babylonian period (1) (and later) knew and solved equations of the second degree (2) -- like this (3):

Obv.11,1. Length and width added is 14 and 40

\[ x + y = 14 \]
\[ x \cdot y = 40 \]

2. The magnitudes are not known. 14 times 14 (is) 3'16 1/4, 40 times 4 (is) 3'12".

\[ 14 \cdot 14 = 196 \]
\[ 40 \cdot 4 = 192 \]

3. 3'12" from 3'16" you subtract, and 4 remain. What times what

\[ 196 - 192 = 4 \]

4. shall 1 take in order to (get) 2 times 2 (is) 4. 2 from 14 you subtract, and 12 remain.

\[ \zeta' \times 4 \rightarrow \zeta = 2 \]
\[ 14 - 2 = 12 \]

5. 12 times 30' (is) 6. 6 is the width. To 2 you shall add 6, 8 is it. 8 is the length.

\[ 12 \cdot \frac{1}{2} = 6 = y \]
\[ 2 + 6 = 8 = x \]

This short text will serve to locate the basic problem of the present paper. Apart from the statements of the problem and of the result, the text contains nothing but the description of a series of numerical computations -- it can be characterized as an exemplification of an algorithm. Even problems 18 and 19 of the same tablet (Metrology 111, 16f), which describe a procedure abstractly, do so on the purely algorithmic level: "Take length, width and diagonal times length, width and diagonal. Take the surface times 2. Subtract the product from the (square on length, width and) diagonal. Take the remainder times one half ...".

There are no explanations of the way the solution is found, no justification of the steps which are made and, so it seems, no
indication whatever of the pattern of thought behind the method.

Now it is an old observation that traditional algebraic problems can be solved by basically different (though often homomorphic) methods. So, if we look at a problem of the type \( x \cdot y = a \), \( x \cdot y = b \), we would of course solve it by manipulation of symbols. Most Latin and Arabic algebras of the Middle Ages, from al-Khwārizmī onwards, would formulate it that "I have divided 10 into two parts, and multiplying one of these by the other, the result was 21"\(^{(5)}\); in order to obtain the solution, they would call one of the numbers "a thing" and the other "10 minus a thing", and by verbal argument ("rhetorical algebra") they would transform it into the standard problem "10 things are equal to 10 dirhems and a square", the solution of which was known from a standard algorithm. Diophantos would speak more abstractly of "finding two numbers so that their sum and product make given numbers"\(^{(6)}\); he would exemplify the method in a concrete case, "their sum makes 20 units, while their product makes 96 units", and he would proceed until the complete solution by purely rhetorical methods, formulated however by means of a set of standardized abbreviations ("syncopated algebra")\(^{(7)}\).

In the so-called "geometric algebra" of the Greeks, geometrical problems of the same structure are solved\(^{(8)}\). So, in Euclid's Data, proposition demonstrated by stringent geometrical construction that "if two lines contain a given surface in a given angle, and their sum is also given, then they must both be given\(^{(9)}\)."
Figure 1. The geometrical figure by which al-Khwarizmi justifies the solution
roots $\sqrt{\frac{10}{2} + 39 - \frac{10}{4}}$
to the problem "a square and ten roots
equal to thirty-nine dirhems"
$(x^2 + 10x = 39)$.

Figure 2. A figure which can serve as
a naive geometric justification of the
computational steps of YBC 34568 NO 9.
the standard algorithms by means of which he solves the basic mixed second-degree equations. To avoid any confusion with the much-discussed "geometrical algebra" I will propose the term naive geometry (10). Since this concept will be fundamental for the following, I shall present it more fully.

In order to justify his solution to the equation "square and roots equal to number", al-Khwārizmī explains the case "a square and ten Roots are equal to thirty-nine Dirhems" (11). The number 39 is represented by a composite figure: A square of side equal to the unknown "Root" and two rectangles of length 5 (±10/5) and width equal to the Root, positioned as shown in Figure 1 (full-drawn line). The gnomonic figure is completed by addition of a square equal to 5² = 25 (dotted line), the whole being then a square of area 39 + 25 = 64. Its side being $\sqrt{64}$ = 8, the unknown Root will be 8 - 5 = 3.

We may feel comfortably sure that the argument behind our Babylonian algorithm was not of the Euclidean brand - Babylonian geometric texts show no trace at all of Euclidean argumentation. We can also safely exclude the hypothesis that the Babylonians made use of symbolic algebra (12). Finally, we can also be sure that some kind of argument lays behind the text. Random play with numbers might of course lead to the discovery of a correct algorithm for a single type of equation, and such an algorithm could then be transmitted mechanically. Still, the equation-types of Babylonian mathematics are so numerous, and the methods used to solve them so freely varied that random discovery cannot explain them. Some mental (and perhaps also physical) representation must have been at hand which could give a meaning to the many intermediate numbers of our algorithm (196, 4, 192, 4, 2, 12, ½) and to the operations to which they are submitted.

We cannot, however, read out of the text whether this representation was of rhetorical-arithmetical character or should be
described as naive geometry. Truly, the "length", "width" and "surface" might seem to suggest the latter possibility. But even Diophantos used a geometrical vocabulary ("square", "application") which was only meant to suggest the arithmetical relations involved; similarly, the Arabic and Latin algebra of the Middle Ages would speak indifferently of a second power as "square" or "property" (and of a first power as "thing" or "root"), intending nothing but suggestive words which might fill the adequate places in the sentences. So, no conclusion is possible on that level.

The procedure leaves us in no better situation. It is easy to devise a rhetorical method which yields the numbers of the text as intermediate results, viz., a verbal translation of this:

\[
\begin{align*}
x + y &= 14; & xy &= 48, \\
(x + y)^2 &= 196; & 4xy &= 192, \\
(x - y)^2 &= (x + y)^2 - 4xy &= 192 - 192 = 4, \\
x - y &= \sqrt{4} = 2 \text{ (the length is normally supposed to exceed the width; hence, no double solution will arise)} \\
2y &= (x + y) - (x - y) = 14 - 2 = 12 \\
y &= \frac{1}{2} \cdot 12 = 6 \\
x &= (x - y) + y = 2 + 6 = 8
\end{align*}
\]

It is, however, just as easy to devise a geometrical figure on which the correctness of the solution and of the single steps can be argued naively—see Figure 2. Here, a geometrical counterpart of every single number occurring in the calculation can be found. So, the algorithm leaves us in a dead end; it fits equally well to a rhetorical argument by arithmetical relations and to an argument by naive geometry.

Concerning another aspect of the question arithmetic/naiave geometry we are no better off than in the case of the method,
namely regarding the conceptualization of the problem itself:
Was it seen as a problem of unknown numbers (represented perhaps by the dimensions of a geometric figure), or shall it be taken at its words, as a problem really concerned with unknown dimensions of such a figure?

That this latter question must be separated from that of the character of the method can be seen from comparison with other algebraic traditions. It is clear that Modern mathematics thinks of a set of equations like \( x \cdot y = 14 ; x \cdot y = 44 \) as concerned with numbers, and that we understand the operations used to solve it as purely arithmetical operations. So, the basis of Modern algebra is arithmetical in conceptualization as well as method.

It is equally clear that we meet with lots of concrete problems, e.g. of a geometrical sort, which we translate into algebra and then solve by algebraic methods. In such cases, our conceptualization is concrete, e.g. geometrical, but our method is arithmetical - concrete entities are represented by abstract numbers.

On the whole, the same description would fit the Medieval algebraic tradition, with one important exception: The al-Khwārizmīan justification of the solution to the mixed second-degree equations, cf. above. There, the conceptualization of the problems is as arithmetical as everywhere else in al-Khwārizmī’s algebra - but the method is naïve geometry, where lines and surfaces represent the abstract numbers. Basic conceptualization and method need not coincide.

To anybody reading Babylonian algebraic sources it will be obvious that the conceptualizations of the problems are as varied as those of Modern algebra. Some are quite concrete geometrical problems - partitions of triangular or quadrangular fields, calculations of the volumes of siege ramps, etc.; some are formulated as
pure number problems, concerned e.g. with a pair of numbers belonging together in a table of reciprocals; the main body of texts, finally, deal with "lengths", "widths" and "surfaces" which cannot a priori be interpreted at face value, nor however as arithmetical dummies. Anyhow, there can be no reasonable doubt that these latter problems represent the basic conceptualization of Babylonian algebra, and that their "lengths" etc. are the entities which represent real lengths as well as numbers when such magnitudes occur in other problems.

There are, then, two main aspects of the problem investigated below. Firstly, whether the method used in Old Babylonian algebra was arithmetical (rhetorical or related) or naive-geometrical. Secondly, whether its basic conceptualization was arithmetical or geometrical. Around these basic questions a web of other related and derived discussions will be spun, in order to give an all-round picture of the discipline.
II. The obstacles

Neither the terminology nor the procedure of the problem translated above would permit us to decide this question, or just to approach it. In this respect, it is similar to a great many other Babylonian texts. For half a century, it has been the prevailing opinion among historians of mathematics that at least the surviving and published texts will not permit us to solve the dilemma arithmetic/geometry. At the same time, most historians have implicitly or explicitly tended to favour the fully arithmetical hypothesis\(^{(15)}\) - with the partial exception of Kurt Vogel, A.A. Vajman and B.L. van der Waerden\(^{(16)}\).

Until the Summer 1982, I shared these common opinions and prejudices (as I would now call them). At that time, however, I was inspired, by an interpretation of a puzzling text\(^{(17)}\) and by a critical question from Peter Damerow for my reasons, to look for traces of geometrical thought in other texts. Since then my knowledge of the language has improved so much that I have come to regard my original inspiration as totally wrong\(^{(18)}\). But like another Columbus I had the good luck to hit land on a course which I had chosen for bad reasons. A close reading of the texts, and the use of methods closer to those of contemporary human sciences (linguistics and structural semantics as well as literary analysis) than to those traditionally used in the history of Ancien
tica/mathematics, revealed that the arithmetical hypothesis cannot be upheld. As it is always more difficult to verify than to falsify, I cannot claim that the investigation has proved a specific geometrical interpretation to be correct; still, the geometrical reading gets very strong support, and I think it can be taken for sure that the Old Babylonian algebra must at least have been structurally isomorphic to a representation by naive geometry, while the
arithmetical representation is only a homomorphism.

It will be clear from the following that my results could not have been found without methodological innovations. So, we should not wonder that the evidence against arithmetical thought has gone largely unnoticed for 50 years, and that the interpretation which Neugebauer characterized as a "first approximation" in 1932(19) has stood unchallenged since then.

This may sound cryptic to readers who are not familiar with the cuneiform script and may require an explanation. The Babylonian texts were written in a Semitic language (Akkadian) which has been dead as a literary language for two millennia (and as a spoken language even longer), with strong, at times all-dominating admixtures of loanwords from another language (Sumerian), which was probably already dead around c. 1800 B.C. except as a literary language used by the restricted circle of scribes, and of which no relative is known. Even the interpretation of the Akkadian language is far from completed, and the situation for Sumerian is still worse(20). To add to the confusion, the script used consists of signs which may stand for one or (normally) several phonetic values (not necessarily close to one another), and for one or (often) several semantic ("ideographic") values, i.e. values as word signs ("logograms") for Sumerian words and semantically related Akkadian words, the connexion between the different values being rooted in semantic affinity, in phonetic affinity in either of the two languages, or simply in the conflation of originally separate signs(22). To all this may come trite problems of legibility, due to careless writing or bad preservation of the tablets.
Happily, the system was also ambiguous for the Babylonian scribes themselves, and they developed certain aids for avoiding the ambiguities (phonetic complements to logograms; semantic determinatives). Furthermore, inside texts belonging to a specific type and period, the range of possible values is strongly restricted. The restrictions, however, have to be discovered; hence, extensive knowledge of a whole text-type is required before the single text can be safely transliterated into syllabic Latin writing.

On this background, the immensity of the task solved in the 1930s by Neugebauer and Thureau-Dangin will be seen: to decipher the phrasing of the mathematical texts, and to discover the mathematical meaning of the terms. First when this is done in a way which can be relied upon can the question of conceptualization be raised in earnest.

Raised ... but hardly solved by direct methods. Just because the language of the single text-types is specific, we must regard the terminology as technical. We know from modern languages that the semantic contents of a technical term is not necessarily unravelled by etymological studies. The etymology of "perpendicular" would lead us to the pending plumb-line and thence to the vertical direction. A posteriori we can understand the way from here to the right angle - but we cannot predict a priori that "vertical" will change into "right angle", nor can we even be sure that a modern geometrener thinks of verticality when he uses the standard-phrase and raises a perpendicular.

The situation is not very different in Akkadian, or in Semitic languages in general. An example from Hebrew on which I shall draw below will show this. \( ^2 \)bq has, as a verb, the meaning "to fly away". Hence we have nominal derivations "(light) dust" and "pollen"
(HAW, 76); from "light dust" probably the tablet covered
with light dust or sand, the "dust abacus", - and from here ap-
parently the "abacus" in general (26). Who would imagine that the heavy
table on which stone calculi are moved was, etymologically, some-
thing flying away?

Truly, the character of Semitic languages is such that the
basic semantic implications of the root from which a word derives
are rarely or never lost quite of sight - they are conserved at least
as connotations. Such conservations are forced upon the users of
the language by its very structure (75). But a requirement that
there should be a semantic umbilical chord between the general and
the technical meaning of a term can at most be used as a control
with hindsight, when the technical meaning has already been
interpreted tentatively. It can tell nothing in advance.

In principle, technical terms should therefore be interpreted
from technical texts. Here, more than anywhere else, the Wittgen-
steinian dictum should be remembered: "Don't ask for the meaning -
ask for the use". Then, however, we are led into a vicious circle:
Our sole access to the use of the technical terms is the body of
texts, which only tell us about the use if we understand their
terms. As long as two conflicting interpretations of the terminology
both permit coherent understanding of use and meaning, neither
can be rejected. And indeed, if we believe in an arithmetical
interpretation of Babylonian algebra, we are led to an arithme-
tical interpretation of the unknown terms describing its opera-
tions, and thus to a confirmation of our initial beliefs; initial
belief in a geometrical interpretation is, however, equally self-
confirming.

Let us take an example, the phrase
10 itti 10 gutškil-ma: t-40"(26).
itti can be translated "together with", and the enclitic particle
-ša by "and then" or "and thus", or it can simply be represented
(as I shall do in the following) by ":". So, the phrase
can be partially translated as

10 ūšēkīl together with 10: 1'40", and so we know that ūšēkīl represents an operation which from
10 and 10 creates 1'40": 100 - either an arithmetical multipli-
cation of pure numbers, or a geometrical operation creating a
rectangle with sides 10 and 10 and a corresponding surface of
100. The form can also be recognized as the imperative of a recipro-
cative causative stem derived from šālam, "to eat", or from
wālim, "to hold" (in which case the transcription ought to be
šēlēlī)\(^{(27)}\). Hence we have the interpretation

_make 10 and 10 eat/hold each other: 100,_
or, if we do not see what "eating" or "holding" has to do with the matter,
and if we want to keep the question explicitly open, we may represent
the semantic basis through a dummy \(\mathbb{M}\):

_make 10 and 10 \(\mathbb{M}\) each other: 100.

In both ways, we get something like idiomatic English as translation
of the phrase. Still, concerning the question arithmetical versus
geometrical interpretation we are no more wise.

Truly, most standard terms of Babylonian algebra look less
opaque than "mutual eating/holding", "to append" \(x\) to \(y\), "to pile
up" \(x\) and \(y\); "to tear out" or "to cut off" \(x\) from \(y\) or to see
"how much \(y\) goes beyond \(x\)"; "to break \(x\) to two"; - all of
these can, as descriptions of additive and subtractive procedures
and of halving, respectively, be interpreted concretely, and all
seem to suggest an imagination oriented toward something mani-
ifest, e.g. the procedures of naïve geometry, rather than an abstract
arithmetical understanding. But so do the Latin etymologies of
"addition" and "subtraction"; like these, several of the Akkadian
terms were established as standard expressions, and some may have been fixed translations of age-old terms. There may have been as little concrete substance left in them as there remains of lead in a right angle.

In the level of single terms and their applications the texts are thus not fit to elucidate the conceptual aspect of Babylonian algebra and mathematics.
III. The structural and discursive levels

I started my search for traces of naive-geometrical thought on
precisely the level of single-term application and literal meanings,
and I was soon able to draw the negative conclusions just presented.
At the same time, however, the close reading of the texts had led me
to some real clues. One of these is the structure of the total
mathematical terminology used in the Babylonian algebraic texts (28).
The other has to do with what could be called the "discursive
aspect" of the texts (as opposed to technical and terminological
aspects): The way things are spoken of and explained, the organi-
zation of explanations and directives, and metaphorical and other
non-technical use of seemingly technical terms (29).

The clues implied by the discursive aspect of the texts can
only be demonstrated on specific examples, and I shall postpone
their presentation. Part of the evidence provided by the structural
evidence can, on the other hand, be explained in abstract form.
Instead of retelling my Odyssey through the texts completely and
from the beginning (30), I shall therefore present some basic
results abstractly before going on to a selection of texts in order
to penetrate further. Exemplifications and supplementary arguments
will be given on the basis of these texts.

In current English, the expressions "a times b" and "a multi-
plied by b" describe the same process—they are synonyms. Which
one to choose in a given situation is a matter of style—as will
be demonstrated by the fact that person A may choose the one in
a situation where person B would choose the other, or that the
choice depends on audience (school children versus mathe-
maticians) or medium (oral or written). We have two different
expressions at our disposal, but we have only one mathematical
concept.
The Babylonians had many multiplicative expressions: šutškulum (whence Sutškil); našūm; ël; nim; ešēpum; tab; a-rä; UL.UL; UR.UR. The matter has, to my knowledge, never been discussed explicitly, but it has been taken for granted and selfevident that all(31) described the same concept(32).

As long as an arithmetical conceptualization was itself taken for granted (and taken for granted to such an extent that the mere possibility of alternative conceptualizations was not recognized), this automatic conflation of all multiplicative concepts was unavoidable: in an arithmetical conceptualization there is only one operation to be described, there can be only one concept(33).

Still, selfevident as it has appeared to be, the conflation is not true to Babylonian mathematical thought. The terms are not synonyms, the choice among them is restricted by other criteria than those of style, taste and dialect.

Truly, some sets of terms are synonyms. ël is the Sumerian equivalent of našūm, "to raise", and it is used logographically in exactly the same functions. nim, Sumerian equivalent of ešē, "to be high" and used even for its factitive stem "to make high", is used instead in a few texts. Similarly, šutškulum (and the logogram ka) is replaced by UL.UL in certain texts and by UR.UR in a few others. But while the choice of a term inside a group is free, the choice of the group from which a term shall be taken is subject to clear rules - rules which in a geometrical interpretation of the procedures are easily stated.
IV. Basic vocabulary and translational principles

Most other classes of arithmetical operations are also subdivided in Old Babylonian mathematical thought, if we are to judge from the Old Babylonian vocabulary\(^{(24)}\). As a preparation for the presentation of the texts, I shall summarize in schematic form the basic vocabulary and its subdivisions, indicating in rough outline the use of each subclass. I shall also give the "standard translations" of the terms which I am going to use in my translations of texts in the following chapters, together with the translations of the terms given in AHw\(^{(35)}\).

IV.1. Additive operations

Two different "additions" are distinguished. The first is described by the term *wagšum* (AHw "hinzufügen"), and it is used when something is added to an entity the identity of which is conserved through the process (the nominal derivative *gibum* designates *inter alia* the interest, which does not change the identity of the capital to which it is added). The Sumerian *dab* is used as a logogram. In order to avoid associations to the modern abstract concept of addition, I use the standard translation "to append" for both terms.

The other addition is designated by *kanšum* (AHw "schichten, häufen"). It is used when several entities are accumulated into one "heap" (cf. the etymology of "accumulation" from "cumulus"), which is identical with neither of them. *gar-gar* and UL.GAR are both used ideographically in the same function\(^{(36)}\), apparently as pure logograms, for standard translations of all three terms I use "to accumulate".

While no separate name for the sum of an "identity-conserving" addition is found (for good reasons, of course), the "accumula-
tion” can be designated by various derivations of kamārum: kinārum, a feminine plural (37) (whence my standard-translation “things accumulated”), nakwartum (standard translation “accumulated”) and kumurrum (“accumulation”). gar-gar and ULaGAR can both serve logographically in the same functions.

IV.2. Subtractive operations

Even subtractions may and may not conserve identity. The “non-conserving” subtraction compares two different entities, by means of the expression mala x eli y liter, “as much as x over y goes beyond” (from malārum, “Übergross, Überschüssig sein/ werden”, with the logograms SI and DIRIG). The most common term for the “identity-conserving” subtraction is namārum, “ausreissen”, with logographic equivalent zi. I shall use the standard translation “tear out”. Another term with the same function (but apparently a slightly different shade) is karārum, “abschneiden” [etc.], st. transl. “cut off”. In specific situations, a variety of other terms may occur.

IV.3. Multiplicative operations

The standard expression of the multiplication tables is “x a-rā y” where x and y are pure numbers. It is also found in a few of the problem texts (normally in double constructions, cf. below). The semantic base is rā, “to go” (cf. Danish gange, “times”, from gå, “to go”, and the analogous Swedish terms). After having used initially the modernizing standard translation “x times y” for “x a-rā y” I have opted for “x steps of y, mainly because even Seleucid texts remember this sense of the term, as revealed by their use of a genitive for the second factor (cf. below, section X.2, BH 34568 No 9).

The term osārum and its equivalent lāb -- “to double” whence even the extension “to make multiple” -- were already mentioned. It is
Figure 3. How to find by “raising” the height of a slope, the number of bricks in a wall, and the area of a field.
used for multiplications of any concrete entity by a positive and not too large integer, and apparently meant as a concrete repetition of that entity. When used to "make multiple", it occurs in phrases like "x ane n nāšum", "to double x until n", or "x a-ra n tab", "to double x n steps" (the deviating use of a-ra will be noticed. In all cases, I use the standard translation "to double", since this basic meaning cannot have escaped any Babylonian mathematician and is inherent even in the sign for tab: ☐.

The third group is made up of nāšum ("(hoch)heben, tragen"), its Sumerian equivalent 11 (the normal logogram for nāšum), and the Sumerian nīm, apparently also used logographically in certain texts. As mentioned above, the latter term means originally "be high", equivalent of Akkadian ešû; perhaps it is used as a pseudo-Sumerogram for the (factitive) 0-stem addû of this word.

These terms are used for the normal calculation of concrete quantities by multiplication: When multiplying by the tabulated constant ("i gī-qub") factors; when multiplying by a reciprocal as a substitute for division (cf. below); in all situations involving a factor of proportionality; and when the areas of trapezoids, triangles, and trapezoids are found. As standard translations I use "to raise" for nāšum and 11 (the alternative "to carry" cannot be brought into semantic harmony with nīm). For nīm I use "to lift".

It is not immediately clear what "raising" and multiplication have to do with each other. A possible clue is provided by the use of the expression "11 sum of 1 cubit (height)" (11 with phonetic complement 11-sum, indicating a derivation from nāšum with ending -sum, e.g.

Fig. 3 nāšum, a substantivized participle meaning "that which raises") as a measure for the inverse gradient of a slope, i.e. the length one has to progress in horizontal direction corresponding to an elevation of 1 cubit.

Figure 3A shows the situation, demonstrating the role of the 11-sum as a factor of proportionality, and at the same time that
it can indeed be seen as "that which raises the slope 1 cubit". Figure 3B shows the same in a less sophisticated manner (used occasionally for that same reason in modern elementary teaching), closer to the Babylonian term that the Greek-type Figure 3A.

"Raising" is also used for two other types of "multiplication" which are testified early in Mesopotamian sources: brickwork calculation and area calculation. That the calculation of the total number of bricks in a wall from the number in one layer can be regarded as an "elevation" of that layer is obvious (interestingly, the term used when a wall is elevated in brick layers is precisely  ullam -- ANw 208.13-14). A comparison of Figure 3B and Figure 3C shows that raising of a slope and "raising" of a wall can easily have been imagined in a common way; Figure 3D, finally, indicates that consideration of a rectangular area as consisting of unit-strips (which is testified by the terminology, cf. below, section VII.2) can have been understood metaphorically in the same way, as the "raising" of 1 strip to the total width of the field.

The last group of multiplicative operations is made up by šatškulum, "to make eat/mold each other", and its various semantic cognates: š-š-kú-kú and š-š-kú (its logograms), UL.UL and UR.UR. (Some further cognates turn up below under the heading "squares"). In the algebra-texts, these terms are only used when an entity which may be considered a "length" is multiplied by another which is a "width", or by itself. That is, in a geometric interpretation of the texts it is used when a rectangle or a square is considered (in fact, as we shall see below, when it is produced). To a modern mind it might be tempting to interpret this as an indication that the term is used for the calculation of an area, since this involves the multiplication of two quantities of dimension length. The falseness of such an interpretation is, however, obvious from the way the areas of triangles, trapeziums and trapezoids are found: As soon as calculated average lengths are multiplied, the term used is našum, šš or nššm.

The interpretation of šatškulum understood as "mutual eating"
is less than self-evident. Truly, an idea which was advanced by Solomon Gandz\(^2\) in order to explain the use of *ukullum*, "ration of food", as a term for the inverse gradient of a slope, could be extended as a last resort: In Hebrew, a field covered by vines is said to be "eaten" by the vines\(^3\). Similarly, a "mutual eating" inherent in *Atskulum* could be read as "mutual covering". To "make length and width cover each other" should then mean "to make them define/confine" a surface -- *viz.*, a rectangulat surface, since it is fully described by length and width. The case where "length and length" are made eat/hold/cover each other\(^4\), on the other hand, turns out to describe the construction of an irregular quadrangle.

It would, however, seem much more obvious to conceptualize the situation as a length and a width (or a length and another length) "holding" between each other the rectangle/in question. In either case the geometrical contents of the metaphor is the same, the two lines confining together a surface. For standard translation I shall use the phrase "make A and B span" (which should be neutral with regard to the two possible derivations though slanted towards "holding"). Two texts (VAT 0390 and A0 8862, cf. below) make explicit that surface construction is meant, telling that "length and width I have made span: A surface I have built".

The ideogram l - kū - kū seems to derive simply from the reciprocity of the St-stem (the form l - kū being a mere abbreviation: it is mainly used in the utterly compact "series texts"). UR.UR and U.L.U.L have the same repetitive structure; their semantics is probably best explained in connection with the concepts for squaring, to which we shall turn next.

As it will be seen below, the term *taktum* (read as *Saktum* inHKT I), which turns up in specific connections during the solution of second-degree-equations, must be related to *Atskulum*; I shall use the term untranslated. Detailed discussions of its meaning and use must await its occurrence in the texts. At present it should only be observed that according to all available evidence it cannot derive from *akšum*, which forms no D-stem. Its close connection to *Atskulum* implies that the same must hold for the latter term (in which case, by the way, the correct transcription will be *Atskółum*), cf. note 27).
IV.4. Squaring and square-root

The two fundamental verbs belonging to this area are si,,
"to be equal", and ashrum, "gegenübertreten (as an adversary,
as an equivalent)" etc. From the mid-third millennium onwards,
si, is used to denote a square as [a quadrangular figure with]
equal sides. At approximately the same early epoch, it is also
seen to denote the equality of the lengths alone or the widths
alone in quadrangles. In the Old Babylonian texts, it is found
with a prefix as šu-si, -- literally a verbal form, probably
meaning "it makes equal". It is used when square-roots are ex-
tracted, at times inside constructions where it stands clearly
as a verb, at times seemingly as a noun identifying the square-
root itself. In YBC 6504 (MKT III, 22f) and in the
"series texts" it is used for (geometrical or arithmetical) squaring
(cf. note 63), and in one text it denotes an indubitable
geometric square.

To a modernizing mathematical interpretation this looks like
primitive confusion: The Babylonians use the same term for a
square (number) and its square root. Such a reading is, however,
anachronistic, due to a pattern of thought which would have looked
confused to a Babylonian: We conflate the geometrical figure
characterized by equal (and mutually orthogonal) sides with
one of its attributes, viz. the area which can be ascribed to it
(the square "is" 25š, while it "has" a side of 5š). The conflate
with the figure with another attribute, viz. its side (the square figure
"is" 10 nindan, while it "has" an area of 1 iku = 100 nindan*).
Following a proposal by Jörn Friberg, I shall use the standard
translation "equilateral" in cases where the term is used as a noun.
This should avoid the wrong connotations following from the use
of words bound up with our own conceptual distinctions and confor-
tions. When the term is used as a verb, I shall use "to make equi-
lateral" - the reasons for this will be given below on the basis of the texts.

Mathematical texts in mahrum itself is mostly used in the sense of "correspond to (as equal)" (this is the standard translation which I am going to use). The derivation mithartum (a nominal derivation, "thing characterized by correspondence/counterposition") is used to denote a square (i.e., as we shall see in the following chapter, a geometrical square) -- once again identified with its side and possessing an area (49). I shall use the standard translation "squared line". The verbal št-stem futamuru ("to make correspond to/stand against each other") is used for the process of squaring (with only one number or length as the object). I shall use the standard translation "raise against itself" (viz. so that a square is formed).

A final important derivative is mahrum (for which /t to be used logographically), "that which corresponds to/stands against its equal". Its function is best explained in connection with occurrences in the texts, so I shall postpone it. As standard translation I use "counterpart".

A number of other terms and signs belong to the same semantic field. Lagab (written kil in Hkt and ins) is used in one text (40) to indicate equality between shares in a field partition; in the "Tell Harmaš compendium" (31) and in one of the Susa texts (32) it denotes the usual square figure ("being" a length and "possessing" an area). With some hesitation I shall treat it as a logogram for mithartum, giving it the same standard translation (33). Ninin (written kil.kil in Hkt) is used in one Susa text exactly as lagab, for the square figure. In the larger part of the Susa corpus it could be replaced by futamuru, as also in some genuine Babylonian texts (35). Finally it is found in a couple of Susa texts with two factors (36), corresponding to the use of futškûlu. This practical equivalence with several semantically related yet glossarially distinct terms makes it impossible to consider it a
real logogram for any of its equivalences; hence, NIGIN is an example of a non-logographic ideogram\(^{(57)}\). Since the sign can replace \textit{lawum}, "umgeben", \textit{mahārum}, "sich wenden","herumgeben" (and its derivative \textit{mīhirtum}, "Umkreis"), I shall propose the standard translation "make surround" (\textit{viz.} surround a square or rectangular figure) and (square or rectangular) "surrounding", depending on the word class required by context.

\text{UR, UR is found in certain texts in constructions similar to those with \textit{Sutakulum}\(^{(58)}\). UR itself is found in another Old Babylonian or early Kassite text\(^{(59)}\) in the sense of "squaring", and in general (non-mathematical) language it can be used logographically (with various complements) for \textit{ištēnum}, "like one", "together" (<\textit{ištēnum}, "one"); for \textit{mīhārtiš}, "correspondingly" (i.e. "equally" or "simultaneously"); \textit{mahārum}, cf. above); and for \textit{nakrum}, "enemy" (probably derived from the association of this concept with \textit{mahārum}, cf. above)\(^{(60)}\). Because of the ideographic but probably not logographic equivalence with \textit{mahārum} I propose the standard translation "oppose".}

\text{UL, UL is found in 7 tablets\(^{(61)}\), in all of which it is used for squarings, in a way which could make it a logogram for \textit{Sutamhurum}. But in one of them\(^{(62)}\) it is also used in the same role as \textit{Sutakulum}, and in another\(^{(63)}\) it is also used as a substitute for \textit{ib-si}, in a situation where this term could be translated "as a square" or "squared", and where it is kept apart from \textit{Sutamhurum} and its relatives. So, we have to do with yet another ideogram to which no well-defined logographic value can be ascribed.

There is no self-evident explanation of the origin of the term. Possibly, it is to be found through homophony: A sound shift between \textit{L} and \textit{R} is possible in Sumerian\(^{(64)}\). Furthermore, UL appears as a rare sign for the sound \textit{ur} (\textit{viz.} \textit{ur},) as well as \textit{ru} (\textit{viz.} \textit{ru},). Since already \text{UR, UR appears to derive from a homophonic shift (from \textit{UR, UR}), a similar shift to UL,UL should not be excluded (especially not since the application of the two terms seems to coincide: mainly squaring but occasionally rectangularization). This is, however, nothing but a hypothesis, and therefore I shall propose a distinct but semantically analogous standard translation, "to confront".}
IV.2. Halving

As it is later seen in Medieval elementary arithmetic, halving is a separate operation in Old Babylonian mathematics, or, rather, it occurs as a specific operation in certain specific connections. Chief among these are the bisection of a side or of a sum of opposing sides when areas of triangles or quadrangles are calculated, and the halving of the "coefficient of the first-degree-term" in the treatment of second-degree equations. The term used is the verb \textit{bequm}, "zerbrechen", in connexions like "break into two" or "half of a break" (where I have used the standard translation "break"). Certain texts use the Sumerogram \textit{gaz}.

The half resulting from a "breaking" operation is designated \textit{bēatum} (occasionally abbreviated or Sumerianized \textit{BA.A}), a term which I shall translate "half-part". It is distinguished from the normal half, \textit{mišum} (\textit{šu-ri-a}), which designates the number \( \frac{1}{2} = 30\)° as well as that half of an entity obtained through multiplication by \(30\°(60)\).

According to parallels from other Semitic languages, \textit{bēatum} was originally a designation for a rib-side or for the \(\frac{1}{2}\) of a mountain ridge. Probably because such a side or slope can be apprehended as one of two opposing sides or slopes, the term is used in a variety of situations where an entity splits naturally or customarily into two parts, or where e.g. a building is composed of two wings. In mathematical texts, it is used similarly for the semi-sum of opposing sides in a trapezium or the semi-diameter of a circle -- all being halves of entities falling naturally or by customary procedure into two "wings".

Below, we shall also see it in an important role in the treatment of second-degree equations (section V.2. on BM 13901 No 1, and \textit{passim}).
IV.6. Division

As it is well known, Babylonian mathematics possessed no genuine operation of division. Division was a problem, no procedure. If the divisor $b$ of a problem $a/b$ was regular (i.e. if it could be written in the form $2^m \cdot 3^n \cdot 5^r$, in which case its reciprocal would be written as a finite sexagesimal fraction), and if it was not too big, $1/b$ would be found in agreement with the table of reciprocals, and $a/b$ would be found by "raising" $1/b$ to $a$. If $b$ was irregular, or if it was too complicated to be recognized as regular, a mathematical problem text would simply formulate the division as a problem, "what shall I pose to $b$ which gives me $a$?" and next state the solution - since normal mathematical problems were constructed backwards from known solutions, the ratio would always be expressible.

Two concepts are important in connection with the method of reciprocals: that of the reciprocal itself, and that of the process of which it is found. The reciprocal of $n$ is spoken of as "igi $n$ qal-bi", eventually abridged to "igi $n$ qal" or simply "igi $n". The literal meaning of the expression is unclear, but it is testified
As early as c. 2400 B.C. in the sense of "the nth\textsuperscript{66}\). Some Old Babylonian mathematical texts use it both in this general sense as regarded as a number "the nth of any quantity" and in the special sense of "1/n"\textsuperscript{7}, but in a way which distinguishes the two\textsuperscript{69}. There is therefore no doubt that the Old Babylonian calculators had a specific concept for the number 1/n, which I shall designate by the standard quasi-translation "ig i of n". The general sense I shall render simply by "the nth part.

To "find" a reciprocal is spoken of by the verb ṣaṭīrum ("ablösen, auslösen"), with the logographic sumerogram du₃, According to Thureau-Dangin\textsuperscript{70} this term should be understood in analogy with the modern metaphor "to solve a problem". However, in two texts the term is also used subtractively\textsuperscript{71}, in a way which is only explained by the literal sense "detach". To "find the reciprocal of n" is thus to be understood as "to detach the nth part[ from 1]"\textsuperscript{72}, a phrase that shall be my standard translation.

The division by an irregular number calls for few terminological commentaries. The term "pose" (my standard translation for ṣakānum-gor, see below) is no term for multiplication; at times, the multiplication to be performed is implicitly understood in the expression, but more often it is stated explicitly\textsuperscript{73}. In the latter cases, the term used belongs invariably to the "raising"-class (maṭānum, 31, nim).

The same was the case when a dividend was multiplied by the reciprocal of a divisor (even when one side of a rectangle is found from the area and the other side\textsuperscript{74}). Apart from the (purely arithmetical) distinction between regular and irregular divisors, division is one thing, and it is the inverse of raising. Nothing corresponding to the distinction between four
different "multiplications" is found. This could be interpreted as evidence in favour of the assumption that the Babylonians understood their division as a common, purely arithmetic inversion of all four multiplications, the isomorphism between which they have of course recognized. Still, since such an understanding would rather lead to use of the purely arithmetic term s-rā, it seems to be a better explanation that the real multiplicative operation was "raising", while the other three classes were in reality something else which could not be reversed (as we shall see below, there are good reasons to apprehend "doubling" as real repetition of the concrete entity, and "spanning" as a constructive procedure; neither of these procedures is of course reversible).

13.7. Variables, derived variables, and units

Besides the 7 operations, a number of basic concepts and appurtenant terms can profitably be presented in advance and briefly discussed. A first group contains the standard names for unknown quantities ("variables"), the way to label new variables, and the units.

By speaking of standard names for unknown quantities I want once more to emphasize that the Babylonians formulated algebraic problems dealing with many types of quantities: Numbers, prices, weights of stones, etc. One set of such unknown quantities, however, belongs with the "basic conceptualization" of Old Babylonian algebra, as unknown abstract numbers represented by letters belongs with our own basic conceptualization (cf. chapter 1).

These basic variables are of course the length and the width. They form a fixed pair. "Length" translates U₆ (very rarely written phonetically with the Akkadian term ṣiddum, "Seite, Rand; Vorhang"). "Width" translates sāg, literally "head, front" (the
rare corresponding Akkadian term is pūtum(75). Both terms appear in surveying texts from Early Dynastic Lagaš (76); surveying is thus a point of origin of the Old Babylonian second-degree algebra (which should not necessarily be confused with its Old Babylonian conceptualization).

Problems in only one variable are basically formulated as concerned with a square identified with its side: nintarum, or LAGAB, NIGIN (see above, section III.4, "squaring and square-root"). In two texts (77), the side of the square is occasionally spoken of explicitly as us, "length", of the "square figure".

In problems in one as well as two variables, the "second-degree-term" is spoken of by the same expression, a-BAB, "field". Like "length" and "width", it is almost invariably written by the cuneiform, but in a number of places it occurs with a phonetic complement indicating a purely logographic use for the Akkadian aqalum(78)(79). I shall use the standard translation "surface" (as I want to avoid the connotations associated with the word "area": A number which describes or measures a surface; such distinction between entity and measuring number is apparently not true to Babylonian thought).

A number of texts use terms like "length", "width" or "surface" for a succession of different numbers (in cases where we would use successively x and 2, etc.). In such cases the two different "lengths" can be distinguished by an epithet appended to one of them: 1ul (CIR in MKI; standard translation "false") or kinum (-gi-na; et. transl. "true"). The use of these terms is best elucidated in connection with their occurrence in specific texts.

In contrast to Modern algebra, the seemingly pure numbers reveal themselves in certain texts as numbers counting a multiple of the basic unit of length, the nīndan (80) (1 nīndan equals
c. 6 m). In problems concerned with volumes, however, the vertical dimension is measured in "cubits" (\textit{ammulum-kus\ü ninda\n}), even when the problem is nothing but "disguised algebra". Areas are measured correspondingly in the unit \textit{sar\ninda\n}, volumes in (volume-\textit{sar\ninda\n-kus\ü}) (i.e. a surface of 1 sar covered to the height of 1 ku\ü).

IV.8. Recording

A large number of terms are used when given quantities and intermediate and final results are announced and taken note of. Some of them are mutually distinct, some are used inside the mathematical texts as "practical synonyms" (although they are not synonymous in their general use).

Most important is \textit{sakoom}, "hinstellen, einsetzen, anlegen; versehen mit", and its numerator \textit{g\,r.} It appears to have a precise technical meaning in the mathematical texts, but since this sense can only be approached by indirect means, I shall use a semantically rather neutral standard translation, "to pose".

The term is often used after the statement of a problem, when given numbers are "posed" before calculations begin - they appear as a preparation for operations to be taken note of in some manner. Similarly, intermediate results are occasionally "posed" (but then mostly "posed to" or "posed by" a length etc. - cf. below). Even a final result can be recorded by "posing"\(^{(82)}\). Finally, it is invariably used in divisions by an irregular divisor, cf. above, section III.6.

The recording of intermediate results can also be spoken of by the verb \textit{lapo\pum}, "eingreifen in, anlassen, schreiben" (rarely, it can also be used for the recording of a given number\(^{(83)}\)).

I shall use the standard translation "to inscribe".

The verb \textit{nado\pum}, "werfen, hin-, niedergehen", is used in two apparently different functions, one of which might look as a practical synonym for \textit{sakoom} and \textit{lapo\pum}. In some texts, when
the "equilateral" (i.e. square-root) of a number has been found, it is "laid down" in two copies, to one of which is added, and from the other of which is subtracted. Two texts use "posing" in the same function, and four employ *kappûnum* in a related way. On the other hand, however, *nadûm* is never used in the other functions of these terms.

The other use of *nadûm* is in the tablet BM 15285, where the drawing of indubitably geometrical squares, circles and triangles is referred to by the term.

Even outside the domain of mathematical texts, similar uses of the term are known: "Bauten usw anlegen"; "(Fang)netz auslegen"; "(auf Tafel usw) eintragen, einzzeichnen"; "Grundriss aufzeichnen"). I shall use the standard translation "to lay down", which shall therefore be read as "to lay down in writing or drawing" (since the former use is restricted to the laying down of entities which in the geometrical interpretation of the texts are the sides of squares, it is my guess that the the real meaning in all mathematical texts is simply "to draw").

A specific phrase for recording of an (invariably intermediate) result is *rēkē likīl*, "may your head retain [it]" (from *rēšum*, "Kopf, Haupt; Anfang, ...", and *kullum*, "(fest)halten"). Apparently, the term is reserved for the storing of intermediate results of linear transformations, cf. below, section VII.2.

The appearance of a result can be announced in various ways. It can be said that a number "comes up for you" (standard translation of *illiakkum*, from *ešûm*, "auf-, esporateigen"); Stative "hoch sein"), or that a calculation "gives" a certain result (my standard translation of *naṣānum*, "geben", and of the Sumerogram *su₂₃*). Finally, the result can be announced by the term *tammar*, "you see" (from *naṣarum*, "sehen"). The choice appears to depend exclusively on the geographical and chronological origin of the text (and in
certain texts perhaps on personal taste). The mathematical functions of all three coincide.

Very often, a result appears simply as a number, announced by no special word or at most by the enclitic particle się appended to the foregoing phrase. A single text uses the Sumero-gram for "posing", g a r -- cf. above, note 82.

11.9. Structuration

The terms discussed till here were all concerned with the "arithmetical" level of the texts, that of single calculations. Another group of terms belongs to the meta-level which makes the texts "algebraic", and which structure the texts.

All those texts which describe a problem together with its solution start by stating the problem, after which the procedure is described. The former is written in the first person (viz. the teacher), past tense (only the excess of length over width will be stated in the present tense). The procedure is formulated in the second person (the student), present tense, or the imperative, by a person (the instructor) who refers to the teacher in the third person. The statement has no special name, but the procedure is designated epēšum with Sumerographic equivalent Ki. The term is an infinitive of a verb ("machen, tun; bauen") used as a noun; when the description is finished, the derived term nēpetum is used. For epēšum I shall use the standard translation "the making", for nēpetum "the having-been-made".

Inside the description of the procedure, the statement of the problem may be quoted as a justification of certain steps made. This is done by the phrase "he has said", using the verb gebūm, "sagen, befehlen", which functions simply as a quotation mark.

A transition from one section of the procedure to the next
may be marked by "I turn around" (from sahārum, "sich wenden ..."; standard translation "to turn around") or "turn back" (from tārum, "sich umwenden, umkehren, zurückkehren"; standard translation "to turn back").

The hypothetical-deductive structure of the complex problem/procedure may be expressed by terms like ānumā ("wenn, falls", st. transl. "if" -- the recurrent term of the hypothetical-deductive sūtra texts), inūmā ("als, wenn usw"; st. transl. "as") and saūmā ("wegen, weil usw"; st. transl. "since"). Most often, it is left implicit -- the statement appears as a fact, and after a phrase (or eventually jussive) "You, by your making" comes an equally descriptive/procedure-part.

The equality necessary to establish an equation is normally implied by the particle -ne followed by a numerical value (the "right-hand side" of the equation) -- cf. above, chapter II (as stated there, I shall render -ne by the sign ":."). If two unknown quantities are equated, the term kīmā ("wie; als, wenn, daß"; st. transl. "as much as") can be found.

A term for equality which may function as sort of bracket is mola ("entsprechend (wie), gemäß"; st. transl. "so much as"), used in the expression "so much as x over y goes beyond", meaning (x - y).

The numerical value of a quantity can be asked for in two ways, either by the question "x minūm" (minūm, "was"; st. transl. "what"; numerographical equivalent en- nam) or by a question like "kt mašī x" (kt, "wie, als, daß"; mašūm, "entsprechen, genügen, ausreichen"; standard translation of the combined expression "corresponding to what"). In a few cases, the student is asked to "make the equilateral [square-root] of x come up" (x hašā-su šuli).
IV.10. The "conformal translation"

Obviously, the shades and distinctions just described in III.1 to III.9 cannot be rendered in a translation, especially not by a translation into a non-Semitic language. One cannot achieve at the same time a one-to-one correspondence for single terms and an acceptable English sentence (not to speak of the rendition of grammatical categories). It is thus for good reasons that Neugebauer restricted the role of the translation to that of a general guide, "selbstverständlich genug genug, um den Inhalt korrekt erfassen zu können, nicht aber, um die Feinheiten der Terminologie und Grammatik daraus ablesen zu können" (89).

Therefore, an investigation of Babylonian mathematics which tries to go beyond mathematical contents and penetrate patterns of thought and conceptualizations must necessarily rely on texts in the original language. On the other hand, the presentation of the results (at least to the non-assyriologist) must by the same necessity approach the question through a modern language.

Since the results of my investigation can only be documented (and partly only explained) with reference to genuine text material, translations are necessary. Since, on the other hand, the translations cannot be allowed to loose those shades and distinctions which cannot be translated into idiomatic English, I have chosen a compromise somewhere between a code and a real translation: All words except a few key terms are rendered by English words; a given expression is in principle always rendered by the same English expression, and different expressions are rendered differently (with the only exception that well-established lexicographic equivalence but distinct typography is rendered by coinciding translations, while possibly more lexicographic equivalence is rendered by translational differentiation); terms of different word class derived from the same root are
rendered (when the result is not too awkward) by derivations from the same root. These translations are the "standard translations" presented above. Furthermore, syntactical structure and grammatical forms are rendered as far as possible by corresponding structure and grammatical forms (the simple style of the mathematical texts makes this possible). Expressed in mathematicians' argot, this sort of pseudo-translation could be called a "conformal translation".

Each line of the translation is followed by a transliteration of the original text. Here, as in current usage, phonetic Akkadian is written emphatically. Sumerian words and Sumerograms (Sumerian words used ideographically for Akkadian speech) are given in spaced writing; and signs which can neither be interpreted one way or the other (either because they should not be, or because our knowledge is insufficient) are written in small capitals. In order to follow the principle of conformity as far as possible, and in order to facilitate the comparison of typographical translation and transliteration, the same/different senses of the same word are used in the translation. So, *kamārum* is translated "to accumulate"; *gar-gar* will be found as "to accumulate" (or another adequate form -- often Sumerograms etc. are found with no phonetic or grammatical complements indicating which grammatical form to choose); and UL.CAR is rendered "to ACCUMULATE". Ideograms written with an Akkadian phonetic complement are translated in mixed writing. So, *a-ṣašum* is translated as "surface". The result violates all ideals of typographical beauty, but it should make it relatively easy for the reader who wants to do so to acquire quickly a rudimentary feeling of the original formulation.

According to analogous considerations, each number is rendered in the translation the way it stands in the original text: Standard
sexagesimal numbers are written in the extended degree-minute-
second-notation described in note 48 (in the transliteration,
the same numbers are given more faithfully, with no indication
of absolute place). Number words, including words for ordinal
numbers and fractions, are rendered by words. Special signs for
fractions are written as modern fractional symbols, \( \frac{1}{4} \), \( \frac{1}{6} \) etc.
Ordinals and fractions written on the tablet as a number followed
by a phonetic or grammatical complement are written 1st, 2nd, etc.

Of course, considerations of intelligibility put some constraints
on the principle of conformity. Prepositions cannot always be
rendered in the same way, nor can some other words which structure
the Akkadian sentences (relative pronouns etc.). Certain details
of the syntactical structure (e.g., postpositive adjective) have
to be given up. Furthermore, definite and indefinite articles
and other English grammatical elements have to be inserted into
the translation. Such insertions stand as normal writing, without
spacing, emphasis and capitalization. In the case of ideograms without
complements even marking of grammatical person etc. are written
that way. Other, genuine explanatory insertions are given as
normal writing in parentheses.

In the transliterations, all restorations of damaged passages are
of course indicated by square brackets. In order not to make
the typographical appearance of the translations too disorganized,
I have omitted there all indications of such restorations, when
they are taken over from the original publications of the texts,
and when I find them firmly established (since the restorations
of HKI, TMB and MCT were made with great care, mainly from
parallel passages of the same tablets, this holds for most
restorations). Restorations for which I am responsible myself
and restorations which I consider more or less dubious are in-
The English terms used as standard translations of Akkadian terms are normally chosen in a way which respects the use of the latter in non-mathematical texts, and which at the same time shows the possible metaphorical use of the term in a mathematical context. A possible alternative would have been a translation by modern technical terms (e.g. "plus" for šamārum, "added to" for waššum, "multiply", "multiply", ..., "multiply," for the variety of multiplicative operations and terms). The point of my choice is not that the Akkadian terms were necessarily used as metaphors and not technically. It is that the technical function of a Babylonian term must be learnt from its own context, not by imposition from the outside of inadequate, modernizing categorizations. Indeed, one need not work for very long with a term like "to append" before one forgets most of the concrete connotations and apprehends its single occurrences technically.

The basic vocabulary for arithmetical operations, for the announcement and recording of given numbers and results, and for the structuration of the texts was presented above together with the standard translations of the single terms. For the sake of clearness, it is listed again in short form in Table 1, where the ordering corresponds to the above discussion. Table 2 lists all terms for which a standard translation is used in the translations of sections V-X, ordered alphabetically according to the standard translations. Table 3 contains the same material, but transliterated ordered alphabetically according to the original language. The latter tables are found just before the bibliography.
<table>
<thead>
<tr>
<th>Akkadian</th>
<th>Sumerian etc.</th>
<th>st. transl.</th>
<th>use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. additive operations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wagšānum</td>
<td>dāl</td>
<td>to append</td>
<td>&quot;identity-conserving addition&quot;</td>
</tr>
<tr>
<td>kanšrum</td>
<td>gar-gar</td>
<td>to accumulate</td>
<td>&quot;identity-cancelling addition&quot;</td>
</tr>
<tr>
<td>kiššitum</td>
<td></td>
<td>things accumulated</td>
<td>sum by kanšrum etc.</td>
</tr>
<tr>
<td>nakmartum</td>
<td>gar-gar</td>
<td>accumulated</td>
<td></td>
</tr>
<tr>
<td>kanmūrrīn</td>
<td>/UL.GAR</td>
<td>accumulation</td>
<td></td>
</tr>
</tbody>
</table>

| **2. subtractive operations** | | | |
| e-ni | ugu...di-tig / Sī | over...go beyond | "subtraction" by comparison |
| matārum | ri | to tear out | "subtraction" by removal |
| kahīnum | | to cut off | |

| **3. multiplicative operations** | | | |
| ṣēgūn | a-rā tab | steps of to double | number times number |
| sašānum | li | to raise | multiplication by positive integer (concrete repetition) |
| šutākulum | nīm | to lift | calculation by multiplication |
| tāšlitum | l-kū(-kū) | to make span | "multiplication" of a "length" by a "width" ("rectangularization") |
| | | tāšlitum | cf. below, sections V.1-2 |

| **4. squaring and square-root** | | | |
| (maššrum) | ib-ṣi | equilateral/to make equilateral | square-root; geometrical square identified with the length of the side |
| šutūnum | | to correspond to | equality of value, shares (etc.) |
| mitartium | | to raise against itself | formation of a square |
| mešgūn | | | |
| | | squared line | square identified with the side |
| | | counterpart | "second side of a square" |
| | | to make surround/surrounding | like šutūnum, mitartium and (rarely) šūtākulum |
| | | to oppose | like šutūnum (and šūtākulum) |
| | | to confront/confronted | like šutūnum, ib-ṣi, (and šūtākulum) |
### 5. Halving

<table>
<thead>
<tr>
<th>Akkadian</th>
<th>Sumerian etc., st. transl.</th>
<th>use</th>
</tr>
</thead>
<tbody>
<tr>
<td>bedu</td>
<td>gas</td>
<td>to break</td>
</tr>
<tr>
<td>bitatum</td>
<td>ba / BA.A</td>
<td>half-part</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bissection</td>
</tr>
</tbody>
</table>

#### 6. Division

<table>
<thead>
<tr>
<th>(igûmu)</th>
<th>igi n (gâi (-bl))</th>
<th>igi of n / n'th part</th>
</tr>
</thead>
<tbody>
<tr>
<td>patârum</td>
<td>du₄</td>
<td>to detach</td>
</tr>
</tbody>
</table>

The fraction 1/n considered as a number / 1/n of something
To find the reciprocal (to take out 1/n from 1)

### 7. Variables, Derived Variables, Units

<table>
<thead>
<tr>
<th>(sûdumu)</th>
<th>uû</th>
<th>length</th>
</tr>
</thead>
<tbody>
<tr>
<td>(pûtum)</td>
<td>saq (-ki)</td>
<td>width</td>
</tr>
<tr>
<td>mithartum</td>
<td>LAGAB</td>
<td>square figure</td>
</tr>
<tr>
<td></td>
<td>NIGIN</td>
<td>surrounding</td>
</tr>
<tr>
<td>(qalam)</td>
<td>a-šâ</td>
<td>surface</td>
</tr>
<tr>
<td></td>
<td>lul</td>
<td>false</td>
</tr>
<tr>
<td></td>
<td>gi-na</td>
<td>true</td>
</tr>
<tr>
<td>kûnum</td>
<td>nindan</td>
<td>nindan</td>
</tr>
<tr>
<td>zamatum</td>
<td>kuû</td>
<td>cubit</td>
</tr>
<tr>
<td></td>
<td>sar</td>
<td>nindan³ / nindan³ , kuû</td>
</tr>
</tbody>
</table>

One of the two basic variables
the other basic variable
the variable in second-degree problems of 1 unknown
- a -
Product, square, and any quantity which in a geometric interpretation is a surface
(optimal) epithet to a length, width etc. different from the one first considered
(optimal) epithet which designates a return to the original use of a term²
Unit of horizontal length, c. 5m
1/12 nindan, unit of height and depth, c. 50 cm

### 8. Recording etc.

<table>
<thead>
<tr>
<th>(Gâkûnum)</th>
<th>gar</th>
<th>to pose</th>
</tr>
</thead>
<tbody>
<tr>
<td>lâpûnum</td>
<td>to inscribe</td>
<td></td>
</tr>
<tr>
<td>nadûnum</td>
<td>to lay down</td>
<td></td>
</tr>
<tr>
<td>râkha likil</td>
<td>may your head retain</td>
<td></td>
</tr>
<tr>
<td>illû-askûn</td>
<td>come up (for you)</td>
<td></td>
</tr>
<tr>
<td>rasûnum</td>
<td>to give</td>
<td></td>
</tr>
<tr>
<td>teamar</td>
<td>igi-du / dû</td>
<td>you see</td>
</tr>
</tbody>
</table>

Presumably material notation and/or drawing, cf. above
Memorization of intermediate results in linear transformations
Announcements of a result
### TABLE 1, continued

<table>
<thead>
<tr>
<th>Akkadian</th>
<th>Sumerian etc.</th>
<th>st. transl.</th>
<th>use</th>
</tr>
</thead>
<tbody>
<tr>
<td>egSium</td>
<td>kl</td>
<td>to make/making</td>
<td>designates the procedure to be used to solve a problem</td>
</tr>
<tr>
<td>nGeSium</td>
<td></td>
<td>having-been-made</td>
<td></td>
</tr>
<tr>
<td>gohüm</td>
<td></td>
<td>to say</td>
<td>designates the procedure when performed</td>
</tr>
<tr>
<td>seharam</td>
<td>nigin(-nal)</td>
<td>to turn around</td>
<td>quotation mark</td>
</tr>
<tr>
<td>törum</td>
<td></td>
<td>to turn back</td>
<td>designates a passage to another section of the procedure</td>
</tr>
<tr>
<td>ūurma</td>
<td></td>
<td>if</td>
<td>- -</td>
</tr>
<tr>
<td>inūna</td>
<td></td>
<td>as</td>
<td>marks a deductive structure</td>
</tr>
<tr>
<td>asēun</td>
<td></td>
<td>since</td>
<td>- -</td>
</tr>
<tr>
<td>kina</td>
<td></td>
<td>as much as</td>
<td>- -</td>
</tr>
<tr>
<td>-m</td>
<td></td>
<td>:</td>
<td>equality</td>
</tr>
<tr>
<td>mala</td>
<td>a-na</td>
<td>so much as</td>
<td>consecution, consequence, result, equality</td>
</tr>
<tr>
<td>mīnūm</td>
<td>en-na</td>
<td>what</td>
<td>a rhetorical &quot;bracket&quot;; equality</td>
</tr>
<tr>
<td>kl magl</td>
<td></td>
<td>corresponding to what</td>
<td>asks for a value</td>
</tr>
</tbody>
</table>

† In a geometrical text (YBC 8633, in HTC, 53), the term "true length" designates that side of a triangle which is closest to being perpendicular to the "width".
V. The discourse: Basic second-degree procedures

As stated in section III, the discursive level of Old Babylonian algebra can only be discussed on the basis of actual instances of this discourse. In the present and the following chapters, I shall therefore present a number of texts, translated according to the principle of "conformity" in order to map the original discourse as precisely as at all possible if the material is not to be presented in the original language. Direct linguistic and philological commentaries are given as notes immediately below the translation of the single texts.

I do not aim at complete coverage of Old Babylonian mathematics. Most practical applications fall outside the scope of the paper, and so do the table texts. The application of the specific methods of Old Babylonian algebra to genuine geometric problems are left aside for later treatment, as are most of the "complex" algebraic applications of the basic techniques. Finally, with a single exception only procedure texts are taken into account: Texts which give nothing but the statement of a problem (or a series of such statements) give little information as long as our understanding of concepts and terminology remains at the present level.

On the other hand, in relation to the class of simple "length-width"-procedure texts the coverage can be regarded as fairly representative. Truly, each new text taken into account brings some new information; still, what is left out appears to me to belong to the category of details and shades, which may await subsequent investigation. The basic features of Old Babylonian elementary "length-width-algebra" can, I hope (and think), be presented adequately on the basis of the present selection of texts.
The problem deals with a pair of numbers belonging together in the table of reciprocals, the igibûm and the igûmûm. The Sumerian forms igi and igi-bi mean "the igi" and "its igi"; they way through the are used most of the text, but a syllabic i-gu-ûm in rev. 5 indicates that the terms are to be read as Akkadianized loanwords though mostly written logographically. Their product (the "surface" of obv. 9) is supposed to be 1' (x=60), or at least an odd power of 60, not 1'. In conformal translation and transliteration, the text runs as follows (to facilitate mathematical understanding, the left margin gives a totally anachronistic commentary in symbolic algebra -- igibûm = x, igûmûm = y):

Obverse

\[ x \cdot y = 60, \quad x - y = 7 \]

1. The igibûm over the igûmûm 7 goes beyond
   [igi - bi] isî igi 7 i-te-ru

2. igûmûm and igibûm what?
   [igi] ù i-gi-bi ni-nu-um

3. You, 7 which the igibûm
   su-te-ma 7 ù i-gi-bi

4. over the igûmûm goes beyond
   ugu igi i-te-ru

\[ \frac{x \cdot y}{2} = 34 \]

5. to two breaks; 3'30'.
   a-na 11-na ki-pa-ma 3,30

\[ \left( \frac{x \cdot y}{2} \right)^{1/2} = 12\frac{1}{2} \]

6. 3'30' together with 3'30'.
   3,30 li-ti 3,30

7. make spon: 12'15'.
   šu-ta-ki-il-ma 12,15

\[ \left( \frac{x \cdot y}{2} \right) \cdot x \cdot y = \]

8. 12'15' which comes up for you
   a-na 12,15 ù i-lic-a-a-kum

\[ \left( \frac{x \cdot y}{2} \right)^{3} = 72\frac{1}{2} \]

9. "the surface append": 1'12'15'.
   [1 a-ta-]2-la-an gi-is-ma 1,12,15
Figure 4. The geometrical interpretation of YBC 6967.
\[
x^2 + y^2 = \sqrt{724} \times 84
\]

10. The equilateral of 1'12'15' what? 8'30'.
   [ib-si] 11,12,15 mi-ru-un 8,30

11. 8'30' and 8'30' its counterpart lay down:
   (8,30 0) 8,30 na-er-3a i-di-ru

Reverse

1. 3'30' the takiltum
   3,30 ta-ki-li-tam

\[
x^2 - \frac{x+y}{2} = 8\frac{1}{2} - 3\frac{1}{2}
\]

2. from the one tear out
   i-na iš-te-en i-su-un

\[
x^2 + \frac{x+y}{2} = 8\frac{1}{2} + 3\frac{1}{2}
\]

3. to the other append
   a-na iš-te-en gi-tu

\[
8\frac{1}{2} + 3\frac{1}{2} = 12
\]

\[
8\frac{1}{2} - 3\frac{1}{2} = 5
\]

\[
x - 12, y - 5
\]

5. 12 is the igungum, 5 is the igungum.
   12 iqi-bi 5 iqi-um

If "going beyond" is interpreted as arithmetical difference,
"breaking" as arithmetical halving, "making span" as arithmetical
multiplication, "surface" as arithmetical product, "equilateral"
as arithmetical square root, and "takiltum" as a factor (in agree-
ment with the interpretation "that which is made span"), most
of this text could agree with an arithmetical interpretation of
Old Babylonian algebra. A few points remain, however, which always
have been seen as peculiar. Why is the "counterpart" of the
square-root introduced? And why are these two copies of the num-
ber 8'30' kept so strictly apart (as a "first" and a "second")
in rev. 2-4?

If a naive-geometric interpretation of the procedure is
made, these two questions are immediately solved, cf. Figure 4:

Since the product of igûm (y) and ighbor (x) is spoken of as a
to be
surface, they are regarded as width and length of a rectangle.
That amount by which the length "goes beyond" the width is
bisection (together with the adjacent part of the rectangle),
and the outer half is moved to a position where it "spans" a rectangle
(in reality a square) together with the inner half. The area of the
resulting gnomon is still 1'. When it is appended to the square
spanned by the two halves (of area \(3'30''\)) = \(12'15''\), we
get a greater square of area \(1'12'15''\). The side producing this
square (or, rather, as we shall see below, the side produced by the
latter area when they are understood as a square figure and thus identified
with its side) is \(\sqrt{1'12'15''} = 8'30''\). It is "laid down" (possibly
"drawn", cf. section IV.8) together with its "counterpart"
(heavy lines). When "that which was made span" the small square (the
\textit{tabiltum}) is "torn out" from the vertical heavy line (its secondary posi-
tion) we get the width (the \textit{igûm}). When it is appended to the horizontal
heavy line (its original position) we get the length (the \textit{igibûm}).

It will be noticed that not a single word of the description
is superfluous or enigmatic when this interpretation is applied.
It can also be noticed that an alternative formulation, the "first"
and "second" \(3'30''\) appended to and torn out from (the same)
\(8'30''\) (e.g. the horizontal heavy line) would be less meaningful,
producing two lines equal but not identical in length and
width.

This sense-making use of "first" and "second" holds throughout the
many texts where they are used. That can scarcely be a random pheno-
menon. So, an interpretation of the doubling of \(8'30''\) as 7 but a
preparation for two different arithmetical calculations can hardly
hold good - in that case, we could expect instances of "first \(3'30''\)
appended to first \(8'30''\), second \(3'30''\) torn out from second \(8'30''\),
and other variations of the same sort. In fact, they are never found.

In other respects too, our text is representative of a whole
group of procedure texts. As already observed above (section IV.8),
the term "to lay down" is always reserved to that process which corresponds to the "drawing of the heavy lines"; if only a number was taken note of for use in an arithmetical calculation, how are we to explain that e.g. the numbers submitted to/"tearing out" are never "laid down"? Similarly, it is a general feature that 3'30' is appended to 8'30'—that quantity which is moved is appended to that which stays in place. The difference is not one of relative magnitude— as we see in obv. 8-9, a greater quantity may well be appended to a smaller quantity; neither is it just a question of fixed habits— when gnomon and square are joined (a situation where both are already in place), either can be appended (94), only where the geometrical interpretation requires that one addend remains in place and one is moved is it apparently impossible to exchange the roles of the two addends. Finally, the concept of a "counterpart" is reserved to roles similar to that which it plays in obv. 11 of the present text; in the case of bisections ("breakings") preparing a purely linear operation it is not used (95).

As we see, all three features are easily explained inside a geometric interpretation. It is, on the other hand, very difficult to find reasons explaining them if an arithmetic interpretation is taken for granted; and it is extremely improbable that the random selection of surviving sources has created a fixed pattern which did not exist originally—our material is not that small.

It will be observed that the text appears to describe a constructive procedure, not argumentation on a ready-made figure like Figure 2. It will also be seen that the procedure coincides grosso modo with that described by al-Khwārizmī (cf. Figure 1).
BM 13901 contains a number of problems dealing with one or
more squares. The first of these is a precise analogon to the
one quoted in Chapter I from al-Khwārizmi. It runs as follows:

Obverse I

\[ x^2 + \frac{3}{5}x = 45' \]

1. The surface and my squared line have
   accumulated: 45'. I the wūṣilum\[\text{wūṣilum}\]
   2-bi\[2\text{bi}\] si (am) \, b mi-it-par-ti ak-niur-ma\[45\,\text{am} \, \text{b mi-it-par-ti}\]
   30' you break 30' and
   2-bi\[2\text{bi}\] si (am) 45\,\text{am} \, b mi-it-par-ti
   30' you make spon.
   ta-ba-ke-an ba-na-at \, tē-hi-pl 30 \, \text{u-s} u-ta-te-kal

\[ x^2 + 2 \cdot 30' \cdot x + (30')^2 \cdot 15' = 45' \]

2. You pose: The half-part\[\text{half-part}\] of 1 you break, 30' and
   30' you make spon.
   ta-ba-ke-an ba-na-at \, tē-hi-pl \, 30 \, \text{u-s} u-ta-te-kal

\[ x^2 + 2 \cdot 30' \cdot x + (30')^2 \cdot 15' = 45' \, 15' \text{to} 45' \, \text{you append: it makes} \, 1 \text{eq} \, 1 \text{lateral}^1, \text{30'} \text{which you have made spon} \]

\[ x^2 + 2 \cdot 30' \cdot x + (30')^2 \cdot 15' = 45' \, 15 \text{am} 45 \text{su-ga-ah-na 1-ia} 1 \text{tē-si} 30 \, \text{u-s} u-ta-te-kal ju \]

3. 15' to 45' you append: it makes 1 eq \, 1 \, lateral^1, 30' which you have made spon

\[ x^2 + 2 \cdot 30' \cdot x + (30')^2 \cdot 15' = 45' \, 15 \text{am} 45 \text{su-ga-ah-na 1-ia} 1 \text{tē-si} 30 \, \text{u-s} u-ta-te-kal ju \]

4. In the inside of\[\text{inside of}\] 1 you tear out: 30' the
   squared line.
   lib-ba \, 1 \text{tē-na-ah-ah-ne} 30 \text{mi-it-par-tum} \]

\[ x^2 + 2 \cdot 30' \cdot x + (30')^2 \cdot 15' = 45' \, 15 \text{am} 45 \text{su-ga-ah-na 1-ia} 1 \text{tē-si} 30 \, \text{u-s} u-ta-te-kal ju \]

\[ x^2 + 2 \cdot 30' \cdot x + (30')^2 \cdot 15' = 45' \, 15 \text{am} 45 \text{su-ga-ah-na 1-ia} 1 \text{tē-si} 30 \, \text{u-s} u-ta-te-kal ju \]

wūṣilum is a nominal derivation from woppin, "heraus-
gehen, fortgehen...herauswachsen...hervorbrechen, herausraggen". The term itself means something going out, including something projecting from a building. Since the mathematical application of the term has never been explained before, I have left it untranslated.

\[\text{half-part}\]

The use of a term for a "wing", a "natural" instead of a mere arithmetical half is noteworthy.

\[\text{half-part}\]

The "makes 1 eq \, 1 \, lateral" translates "1 \, tē-si\, 30'\, \text{u-s} u-ta-te-kal\, ju". The use of the "injunctive suffix" -e (which occurs commonly in this connection) appears to indicate not only that the verbal character of the term tē-si, is still present to the Old Babylonian
calculator, but also that the first "it" is considered the agent of a transitive verb, while the second "it" must be seen as the object. Cf. Thureau-Dangin (1956a:3) note 3, which also quotes an instance of the phrase "mi-um i-l-b s-i-a," where a square-root is asked for; here, too, the square-root must be the object of an act since it is asked for in the accusative. (So also the Suna and most tell Harrai texts).

A number of other texts, however, ask for the square-root by the phrase "i-l-b s-i-a, x mi-um um" (e.g., YBC 6957, obv. 10) or "i-l-b s-i-a, x en-nam" (e.g., VAI 0390, passim, and VAI 8520, obv. 20, rev. 19). mi-um um is an indubitable nominative; in the latter texts, the other occurrences of en-nam are indubitable nominals, while corresponding accusatives are written phonetically as en-nam. In such cases (and when the term is used in the generalized sense of "solution" to an equation), i-l-b s-i-a must apparently be read as a noun, and I shall translate "the equilateral of x how much".

In a few late OB and in one early northern text, the alternative term ba-si-a, originally a verb too, has been adopted into Akkadian as a loanword ba-si-a, which is regarded completely as a noun - cf. IN 52301, No 2, note 6 (below, section X.1).

Following Thureau-Dangin (1956a:31 note 4) I interpret i-lb-ia (SBLA) as i-lb-ia, the construct state of a locative accusative. The other possible reading, SBLA=il-labb, BA=ba (vb) sa (possessive suffix plus locative suffix), would lead to the translation "in its inside", which connects rather badly to the ensuing "it" (the lack of an intermediate s-g forbids an interpretation as an emphatic genitive construction).

The other, equivalent form used in the tablet, libbi, I translate "(the) inside of"

We observe that the "squared line" is in fact identical with the side of the square, while the area of that figure is spoken of by a separate concept, "the surface".

When this usage is accepted, the procedure is grosso modo mapped by the arithmetico-symbolic interpretation in the left margin. However, it remains fully unclear why the number 1 should be spoken of as something "projecting" or "going away". Another puzzle is the choice of the term ba-si-a, "half-part", when the normal term mi-si-a, "half", is used at all places in the tablet where one entity is the half of another entity.

If we try a geometric interpretation, the intention of both

Fig. 5 terms can be made clear -- see Figure 5.
Figure 5. The geometrical interpretation of BM 13901 no 1.

Figure 6. The geometrical interpretation of BM 13901 no 2 (distorted proportions).
As in al-Khwārizmī, a geometric summation of a square and a
tube having the dimension of number 2 sides requires that the number 2 be understood as a
length. This is shown in the first step of the figure, where the "squared
line" is represented by the area of a rectangle of length 2 and
width 1. The figure makes it immediately obvious that the number 2 is something which projects. The only question which is left
open is whether it projects from the square or from the width 1
(as we shall see below, the latter possibility must be preferred).

From here, the procedure is exactly parallel to that of YBC
6967 and Figure 4. Comparing the two texts we can even see why
the need for the term mâsītum arises: while
the problem
of two unknowns could speak of that by which "x goes beyond y",
the corresponding geometrical quantity ("that by which x+1 goes
beyond x") has no obvious designation in the problem of one unknown -- if
not, precisely, mâsītum. This is then posed and next "broken"
(i.e. bisected), and the outer half is moved so that a square is
spanned. This square is appended to the gnomon resulting from
the preceding manipulations of the figure, in order to produce
another square. The side of this great square is found (lit-
early: the result of the appension produces 1 as "equilateral"
finally, the quantity which spanned the complementary square is removed ("torn out"), and the unknown side of the original
square (the original "squared line") is left.

Concerning the "half-part", the situation in the figure is
evidently related to the origin of the term. By the very nature
of the problem, the appended rectangle consists of two "wings",
of which one is to be broken off and moved.

According to both Thureau-Dangin and Neugebauer, the
tablet belongs together with AO 8062 to the oldest stratum of
Old Babylonian mathematics (98). Goetze’s linguistic analysis
ascrives to both a southern origin, probably Larsa (99).
The second problem of the tablet subtracts a side instead of adding it. The text runs as follows:

**Obverse I**

\[ x^2 - x = 16.30' \]

5. _My squared line inside of the surface_  
   I have torn out: 14'30'. 1 the \( \text{m} \)\( \text{s} \)\( \text{g} \)\( \text{t} \)\( \text{um} \)  
   mi-it-bar-ti lib-bi a - ā a (as-su-ub-ma 14,30 1 wa-qil-tum

\[ \frac{1}{2} \times 30' \]

6. _you pose_. The half-part of 1 you break, 30'  
   and 30' you make spon;  
   ta-ša-ša-an ba-ma-at 1 te-ul-pl 30 ā 30 tu-ul-ša-kal

\[ \frac{1}{2} \times 30' \times (30')^2 = x^2 - 2 \times 30' \]

7. 15' to 14'30' you append; 14'30'15' makes  
   29'30' equilateral.

\[ x = 29'30' = 30' \]

8. 30' which you have made spon to 29'30'  
   you append; 30 the squared line.  
   30 ā tu-ša-ša-li  a-ne 29,30 tu-ša-ma 30  
   mi-it-bar-tum

Once again, the text is grossly mode mapped by the arithmetico-symbolic interpretation. Only the problem of the "1 which projects" is left open, together with the question why only the "coefficient" of the first-degree term is "posed", and the choice of the term "half-part".

If the imagery inherent in the terminology ("appending", "tearing out", "breaking", "making spon") is taken at face value, we are led to a geometric procedure which solves even these problems -- see Figure 6. From the square, a rectangle of length \( x \) and width \( s \) is removed. The area of the remaining rectangle is 14'30'.

Since the length of this rectangle exceeds the width by \( s \), a strip of this width is bisected, and its outer wing is moved so as to transform the known area into a gnomon. The small square spanned by the two halves of the strip is appended, and so we get
a square of known area. Its side is found, and the half-strip which was moved in order to span the small square is appended again. This gives us the original length of the rectangle, and thus the side $x$ of the square.

The geometrical procedure is of course the same as that of Figure 4 and Figure 5. The area of a rectangle is given, together with the difference between its length and its width. The excess of length over width is bisected, and the rectangle is transformed into a gnomon, for which the area and the side of the lacking square are known. The area of the lacking square is then found and added to the gnomon, transforming it into a square of known area. The side of this square is calculated, and the original length (Figure 6), width (Figure 5) or both (Figure 4) can finally be found. Indeed, the only difference between the cases (as seen from the geometrical interpretation) concerns the entity asked for.

It is still not to be seen whether the wajitum should be understood as that width "$y$" which must project from the length in order to transform it into an area which can be torn out, or perhaps the excess of rectangular length over rectangular width. In any case, it has a definite role to play in the procedure (and as stated above, the former possibility will turn out to be correct). So, the question, why only the coefficient "$y$" to the linear term is posed, disappears in the geometric interpretation - the wajitum is no numerical coefficient.

So, once again, the arithmetico-algebraic interpretation allows us to understand the main mathematical progress of the calculation but not the details of the formulation; the approach through naive geometry, on the other hand, allows us to understand both the mathematical progress and the discursive organization of the texts.
The three previous problems presented the standard way to solve the basic mixed second-degree equations. The present one exemplifies that the Babylonians would sometimes leave the standard methods.

The problem adds the four sides of a square to the surface - not 4 times the side, but explicitly the four sides:

Reverse II

11. The surface (of) the four fronts and the surface\textsuperscript{*} I have accumulated; 41'40" a-\textit{bālam}pla-(at er-bā-et-timp) \textit{u a-\textit{bālam}} sk-mur-ma 41,40

12. 4, the four fronts, you inscribe. The digit of 4 is 15:

\textit{a-nā} 41,40 \textit{ta-la-pa-at} \textit{gāl-bi} 15

\textit{\frac{5}{2}} x^2 + x =10'25"

13. \textit{\frac{5}{2}} x^2 + x = 10'25" you raise; 10'25" you inscribe.

\textit{15 a-nā} 41,40 \textit{ta-la-pa-at} \textit{gāl-bi} 15

\textit{(\frac{5}{2}x+1)}\textsuperscript{2} = 10'25" + 1 = 1'10'25"

14. \textit{1 the wāqštim you append;} 1'10'25" makes 1'5' \textit{equilateral}.

\textit{\frac{5}{2}x+1} \textsuperscript{2} = 10'25" + 1'5"

15. \textit{1 the wāqštim which you have appended}

\textit{you tear out}; 5' to two.

\textit{1 \textit{wāqštim}} \textit{tu-\textit{la}-\textit{ba} \textit{ta-nā} \textit{sa-\textit{lam} 5 \textit{a-nā} \textit{gī-na}}}

\textit{\frac{5}{2}x = 1'5' - 1 = 5'}

16. \textit{you double}; 10' nūndān stands against itself\textsuperscript{*}.

\textit{tu-\textit{gi-ip-ma} 10 \textit{nu} \textit{ndān} in-\textit{ta-ba-ar}}

\textsuperscript{*} This passage calls for several commentaries. It must be transliterated "a-\textit{bālam} pla-(at er-bā-et-timp) a-\textit{bālam}", where the restitution follows from line 12. Both Thureau-Dangin and Neugebauer prefer the lower possibility, and read "A surface. The four fronts and the surface I have accumulated." This, however, neglects that the initial \textit{a-\textit{bālam} is an accusative (and is pointed out deliberately to be an accusative singular by the grammatical complement \textit{fam}). So, as far as I can see, the only possibility to make grammatical sense of the construction is the upper restitution, which makes the initial surface part of the object of
the "accumulation", governed by a genitive "of the four fronts".

The "fronts" translate plz, plural (construct state) of pilzu. This word is often considered as equivalent of aq, my standard translation of which is "width". Only extremely few texts, however, use the Akkadian word instead of the Sumerogram, none of which belong to the category of standard "length and width"-problems (see above, note 75). Even occurrences of the Sumerogram with an Akkadian phonetic complement are strictly absent. So, the use of the term pilzu in our text must intend something explicitly different from the technical concept "width" - hence the use of the literal translation "front".

The numeral "four" is in status rectus and postponed. This literary stylistic figure appears to belong to situations where the number is an invariable epithet, i.e. where n items belong invariably together ("the seven mountains", cf. ONG § 1391) -- whence "the four" instead of "the four".

The term is istuka (or possibly istukar, the preterite form), št-stem of sumérka, "to correspond to". I deviated from my standard translation of the št-stem in order to be in agreement with the št-stem, "to face against itself".

The term is of course the verbal correspondent of šúršukum, "squared line". The translation reveals how difficult it is to make a real "conformal translation" as soon as unrelated standard translations.

This time, the arithmetico-algebraic interpretations leads into real trouble. Indeed, if a "square" is only a second power, there is no reason to speak of the four fronts (or widths); neither is there any reason to leave the normal concept of the "squared line" for that of "front", nor to ascribe a "surface" to the fronts.

Of course, an arithmetical interpretation can map the mathematical procedure. But it offers no explanation why normal/and procedure are given up in this specific case; in fact, the deviation is so astonishing that Neugebauer suspected it to have arisen by a combination of mistakes which happen to make sense on the tablet.

Finally, the place of the problem (among the complicated variations and not among the simple cases of one variable) is an enigma; so is also the "doubling to two" in a place where an arithmetical interpretation would expect a "raising" (cf. the problem discussed immediately below).
Figure 7. The geometrical interpretation of BM 13901 no 23.

Figure 8. The geometrical interpretation of BM 13901 no 3.
The geometric interpretation, especially as it is made clear by the term wāšītum, solves all these problems--cf. Figure 7. (being itself the squared line) First of all it is clear that a geometric square possesses four sides, which can be regarded as "fronts". Moreover, if we take the text at its words and add four rectangles of length 7 and width x, the "surface of the four fronts" (instead of one rectangle of length 4 and width x, or two of dimensions z times x, as we would normally expect), we get a geometrical configuration which differs from the normal square-plus-sides dealt with in the beginning of the tablet--and thus a reason that the problem is listed among the complicated variations.

The occurrences of the wāšītum confirm that the cross-form configuration is indeed thought of: If we follow the text, we can imagine that the multiplication by 4 in lines 12-13 is a quartering, as shown in the second step on the figure. At first, this is of course only a possibility. In line 14, however, the wāšītum is appended, i.e., not any number but a square 7' identified with the wāšītum; such a square is shown in the third step, where it completes the quartered cross as a square. No other configuration than the cross would allow so literal a reading of the text,- and since the occurrence of the wāšītum in line 14 does not refer to any earlier occurrence, it must refer to the entity itself, not to anything obtained from or equal to the "projection".

In the next step of line 14, the side of the completed square is found, and the same wāšītum is torn out--this rules out Thureau-Dangin's conjecture, viz. that the term may simply fix the order of magnitude to 7' (one need not fix the order of magnitude of a number which is identified with a number previously used), and it confirms that the square which was appended in line 14 is identified with its side: if a squaring of "7" had been left out
by error in line 14, the invariable epithet would have been "which you have made span" instead of "which you have appended" - cf. problems No 1 and 2 from the tablet as quoted above.

The tearing-out of the wasitum leaves half the side of the square (in the right position). It is "doubled" to two, i.e. repeated concretely (002), in agreement with the situation of the figure, giving us one of the fronts. It is, however, not spoken of as a "front", nor designated by the normal term "squared line" (milbartum). Instead, it is stated that 10' is that which "stands against itself" - presumably because no "squared line" was spoken of explicitly in the statement of the problem; instead four "fronts" have been supposed to "stand against each other as equals".

Curiously enough, al-Khwarizmi uses the same figure as an alternative argument for the solution of the problem "square and 10' roots equal to number" (cf. above, section 1). Instead of distributing the rectangle 10'x as shown in figure 1, he distributes it as four rectangles 24'-x along the four edges of the square (003).

The text brings us somewhat closer to the precise meaning of the wasitum. It cannot be the excess of rectangular length over rectangular width. Possibly, it could be the length of any of the four projections from the central square; that would, however, agree poorly with the use in problem 2 of the tablet (see above). So, we are led towards the interpretation of the wasitum as that projecting width "r" which transforms a length into a rectangle of equal area; such a concept seems also to be implicitly presupposed by the expression "surface of the four fronts".
The above problems can all be classified as "normalized mixed second-degree equations". The present problem shows the normal Old Babylonian way to deal with a non-normalized equation. The text runs as follows:

Obverse I

9. The third of the surface I have torn out:
the third of the squared line to the inside
ša-lu-ú-dī ša ša-su-eš-ur-ma ša-lu-ú-dī
mi-it-her-tim a-na 1ib-bi

10. of the surface I have appended: 20°.
1 the wūgitum you pose
a-sālim u-gi-ib-ma 20-e 1 wa-gi-tam ta-ša-ka-en

11. The third of 1 the wūgitum, 20° you tear out:
40° to 20° you raise;
ša-lu-ú-dī ša-gi-tim 20 ta-na-sa-aḫ-ma 40 a-na
20 ta-na-ši

12. 13°20′ you inscribe. The half-part of 20°,
the third which you have torn out
13,20 ta-la-pa-at (ba-ma-at 20 ša-lu-ú-tim ša ta-u-bu

13. you break: 10° and 10° you make spon. 1′40′
to 13°20′ you append
š-bi-pl 10 ša 10 tu-uš-ta-tal 1,40) a-na 13,20 tu-uš-ab

14. 15° makes 30° equilateral. 10° which
you have made spon in the inside of 30°
you tear out: 20°;
15-e 30 (š-bi 10 ša tu-uš-ta-ti-la 11ib-be 30)
ta-na-sa-aḫ-ma 20

15. The 40° of 40°, 1′30′ to 20° you raise:
30° the squared line.
š-bi 40 gāl-bi 1,30 a-na 20 ta-na-ši-ma 30
mi-it-her-tum

* Both for mathematical reasons and because of the many parallel passages of the tablet, this "have torn out" must be a writing error for "you have appended", tu-uš-ba.
The problem is of the type $ax^4 + bx^2 + c = y$. In Medieval (Arabic and Latin) algebra, such an equation would be normalized as $x^4 + \beta x^2 + \gamma = 0$. The method here is different, a fact which has often been regarded as astonishing, although the same procedure is used by Diophantus and Heron of Alex. Instead of $x$, $ax$ is taken as the quantity looked for, and the equation is transformed into $(ax)^4 + \beta (ax)^2 + \gamma = 0$. In the end, $x$ is found from $ax$ through multiplication by the reciprocal of $a$.

The application of the arithmetical interpretation raises a problem: The multiplications by $a$ and $a^{-1}$ are expressed by means of the term "to raise", while that of $(\beta/2)$ by $(\beta/2)$ (of 10 by 10') is expressed by "making span". Another problem is presented through the way the equation is transformed: As most of us would immediately feel, and as it is confirmed by the Medieval algebras, the reduction to normalized form is easier to keep track of in a rhetoric-arithmetical representation than the actual "change of variable". Finally, of course, the wasitum remains a stranger to any arithmetical interpretation, as does the distinction of a "half-part" from a "half".

As usual, we shall try to apply a representation by naive geometry-- see Figure 6. If we look at lines 12-14 of the text, it is clear that they follow the normal "square-plus-sides"-procedure (cf. section V.1 and Figure 5). So, we must interpret the text geometrically in such a way that this situation comes about.

Line 9-10 states the problem. In line 10, furthermore, the wasitum is "posed"-- and since no "projection" from the square is 1, we can now be sure that the term designates that projection from a line which creates the rectangle of equal area, as suggested above. An area of one third of the side is then a rectangle of width "the third of $i$ the wasitum", i.e. 20', and length $x$. This corresponds to line 11-- where, however, an ellipsis turns up, as
the third of the \textit{wágítu} is identified with the third of the surface) which is to be "torn out" (that such a confusion is really there is confirmed in line 12). So, the "coefficient to \(x^4\) ("\(y\)") is found to be 1'-20"=40".

In the last part of line 11, this factor is applied to the total non-shaded area \(\sqrt{\frac{1}{4}x^4}+\sqrt{\frac{1}{4}x^2\cdot20}\). This can be apprehended geometrically as the first transformation of the figure, where the scale factor 40'-f in \(\sqrt{\frac{1}{4}}\) is applied in the vertical direction. This operation transforms the rectangle \(x\cdot\sqrt{\frac{1}{4}}x\) into a square \(\frac{1}{2}x\cdot\frac{1}{2}x\). At the same time, the appended rectangle \(\sqrt{\frac{1}{4}x^2}\) is transformed into a rectangle \(\sqrt{\frac{1}{4}}\cdot\frac{1}{2}x\). That is, we have obtained the required situation "square-plus-sides", and the number of "sides" is unchanged. The rest of the procedure is by now well-known: The appended rectangle is bisected and moved so as to "span" a square of area 1'-40". This area is appended to the gnomon, the area of which is 40'-20"=13'-20". The area of the resulting square is 15', and its side therefore 30'. From this, the side 10' of the square which was "spanned" is "torn out", leaving 20' as the side of the square \((\alpha\cdot)x\). Hence, \(\alpha\cdot= 20'\) and \(x\) itself is found through division by the scale factor 40' (i.e., through multiplication by its inverse 1'-30") to be 30".

This solves all the problems raised by the arithmetical interpretation. First of all, it is clear that the multiplication by a scaling factor or its inverse is different from the geometrical process "to span a square". If the conceptualization and method of Old Babylonian algebra are geometric, a terminological distinction between the two is next to obligatory.

Next, the geometrical interpretation leads us to prefer the "Diophantine" to the "Medieval" reduction: If the non-shaded part were to be transformed into a "square-plus-sides" through
Medieval reduction, the change of scale would have to be in the horizontal direction. This would affect the width of the appended rectangle, which goes into the further calculations; on the other hand, the "Diophantine" transformations affects only its length which is anyhow irrelevant (105).

Finally, of course, the wāṣiṭum is no stranger but a must for a geometrical interpretation (with or without a name), and the "half-part" is a natural half, a "wing".

On the other hand, the geometrical interpretation raises two new questions. The first of these concerns the semantic range of the term "raising": Is it restricted to multiplications which can be regarded as changes of scale, or is it wider? This cannot be answered from the present text, but as discussed above (section IV.3) the range is indeed much wider. Cf. also below, section V.8.

The second question concerns the figure: Did the Babylonians draw or imagine a series of different diagrams, as they are shown in Figure 8? Or were they able to conceptualize the same representation first as a rectangle with sides $x$ and $\frac{1}{2}x$, and next as a square with both sides equal to $\frac{1}{2}x$? It is equally impossible to answer this second question on the basis of the present text (or to give a definitive answer on the basis of any text I know). Yet, as I shall argue in chapter VI, indirect evidence suggests that the Babylonians were fully able to conceptualize a drawn rectangle as a diagram for a square.

The geometrical technique which appears to be used in the first examples (and in al-Khwārizmī's justification) can be described as a "cut-and-paste"-procedure. The same technique is used in the present examples for those operations which are described by the terms "to tear out", "to append", "to break"
and "to make span". The "raisings" of line 11 and 15, however, belong with another technique, of which special notice should be taken: A technique of proportionality, which in relation to uni-directional
the geometric representation can be described as a "change of scale"; I shall use the term "scaling" for the technique.(106)

The above examples were all concerned with mixed second-degree equations. We shall now turn to homogeneous problems - first to BM 13901 No 10.

Obverse II

11. The surfaces of my two squared lines
I have accumulated. 21'15'.

12. Squared line to squared line, the seventh.*
It has become smaller.
mi-it-bar-tum a-na mi-it-bar-tim si-bi-a-tim is-ti

13. 7 and 6 you inscribe. 7 and 7 you make span, 49.
7 6 ta-la-pa-at 7 7 tu-uš-ta-šal 49

14. 6 and 6 you make span, 36 and 49 you accumulate:
6 6 tu-uš-ša-šal 36 49 ta-ká-mar-na

15. 1'25'. The and of 1'25' is not detached.
What to 1'25'
1,25 1 1,25 1 25 ip-pa-ša-an a-na 1,25
lú-uš-ku-un a 21,15 ina-di-nam 15-e 30 š-b-si,

16. shall I pose which 21'15' gives me? 15'
makes 30' equilateral.
lu-uš-ku-un a 21,15 ina-di-nam 15-e 30 š-b-si,

17. to 7 you raise: 3'30' the first squared line.
30 a-na 7 ta-na-es-i-ma 3,30 mi-it-bar-tum 18-ti-a-at

18. to 6 you raise: 3 the second squared line.
30 a-na 6 ta-na-es-i-ma 3 mi-it-bar-tum ša-ni-tum.

The form is a plural, mšištíšim -- cf. Thureau-Dangin 1934:49, and Goette 1946:206.
A geometrical interpretation of the procedure is shown in Fig. 9. The first step, that of finding the set of proportionate numbers, looks like a purely arithmetical "single false position": A number from which one seventh is easily taken away is 7, and the removal of the seventh leaves 6 (107). These numbers are "inscribed"—an expression which was also used in No. 23 and No. 3, where the areas found by quartering and scaling were "scribed". In agreement with Babylonian habits as expressed in tablets with drawings (108), we may imagine inscriptions along the edges of squares, as shown on the figure. This can be so interpreted that a unit is imagined in which the lengths of the squares are 7 and 6, respectively (such a conceptualization could follow as an extrapolation from common experience with metrological conversions). The respective areas are found (by "making span") in the square of this unit, as 49 and 36; the total area when measured will then be $49 \times 36 = 1.75'$. In the basic area unit it is known to be $21'15'$. So, the square of the imagined unit (the area of the small squares) is $21'15'/1'25' = 15'$; hence its side will be $\sqrt{15'} = 30'$, and those of the two original squares $7 \cdot 30' = 3'30'$ and $6 \cdot 30' = 3'$.

Fundamentally, this conceptualization subdivides the squares of the problem directly. An alternative interpretation could be that two auxiliary squares are imagined, of "real" sides 7 and 6. Their areas are found and added; the ratio between this and the original total area is calculated, etc.

It is impossible to decide from the text which interpretation to prefer (from the viewpoint of mathematics, they are of course equivalent). My intuitive feeling is that the former is the more plausible, as it is conceptually simpler—it is easier to draw the subdivisions of an existing square, to point to it and speak about it, than to make non-mathematicians understand an abstract ratio (and the reason why its square-root should be taken). As
Figure 9. The geometrical interpretation of BH 13901 No 10.

Figure 10. The geometrical interpretation of VAT 8390 No 1.
we shall see in the following examples, there is also direct
evidence that the Babylonians used subdivisions and alternative
"units" rather than ratios.

In any case, the text presents us with a third technique
besides the cut-and-paste procedures and the scaling: The calcula-
tion of total "coefficients" -- here the "number of small squares"
(below, we shall meet in section VII.3, TMS XVI, the expression "as
much as there is of" entity x, as an explicit formulation of this
concept). We notice that the number is found by "accumulation",
not by "appending". The same holds for the calculation of the
total area in the lower cases, indeed, none of the
addends possesses a "identity" which is conserved through the
process. It seems plausible, too, that "accumulation" is a more
genuinely arithmetical process than "appending", adding also
measuring numbers, while "appending" affects only concrete though
measured entities.

In order to point to a practice with which the Babylonians were
utterly familiar, and which is structurally analogous to the
accumulation of a coefficient, I shall speak of the "accounting

V.7. BM 15205, No. 10 (HKI I, 130; HKI II, Plate 6)

BM 15205 is (part of) a large tablet where the areas of various
subdivisions of a square of side 1 are asked for. The present
problem is clearly related to a particular aspect of the argument
of the previous problem, and it can serve to elucidate the questions
left open there.

The text is accompanied by a figure, which I show in the
left margin (traced after the photo in HKI II).
1. The length, a squared line
   \( u\hat{s} \; mi-il-ta-ar-tum \)

2. In its inside, 16 of a squared line
   \( saq, -ba \; 16 \; mi-il-ta-ar-tum \)

3. I have laid down. Its surface what?
   \( ad-di \; a-\tilde{a}-bi \; en-nam \)

* The form is a genitive singular.

The figure shows us precisely the subdivision of a square into smaller squares which was suggested as the first interpretation of the procedure of the previous problem. So, this interpretation is at least strengthened.

Another interesting point is the use of the singular genitive in line 2 (true enough, Sogg's suggests\(^{110}\) that we have to do with a simple writing error, but that appears to be excluded by the singular -bi in line 3). The small squares appear to be regarded as repetitions of an identical entity—a unit of accounting. Even in this respect, the present and the previous text are related.

V.B. VAT 8390, No. 1 (MKT 1, 323 1/2, cf. MBA I 172 1/2)

A final homogeneous second-degree problem is VAT 8390, No. 1\(^{111}\),

Obverse I

\[ \begin{align*}
\text{Length and width I have made} & \text{ span: } 10'' \text{ the surface} \\
\text{[u\hat{s} a saq] uta-ta-il-ma} & \text{ 10 a-\tilde{a}} \\
\text{x}^2 = 9 \cdot (x-y)^2 & \\
\text{Length to itself I have made span:} \\
\text{[u\hat{s} a] na re-smi-\tilde{a}-\tilde{u} uta-ta-il-ma} \\
\text{A surface I have built} \\
\text{a-\tilde{a}} & \text{ sb-ni} \\
\text{So much as the length over the width} & \text{ goes beyond} \\
\text{[as]-la u\hat{s} a-pa saq i-te-ru} 
\end{align*} \]
5. I have made spon, to 9 I have doubled:
   \( u₃-th₃-a₃-n₃₃ a₃-n₃₃ 9 \ e₃-a₃-i₃₃-n₃₃ \)

6. As much as the surface, which the length
   by itself
   \( k₃-n₃₃ a₃-a₃₃ ₃ \ a₃ u₃₃ \ i₃₃-n₃₃₃₅₃₃ u₃₃ \ i₃₃-n₃₃₅₃₃-n₃₃ \)

7. has been made spon.
   \( u₃₃-t₃₃(₃)₃₃ \ a₃ \ i₃₃ \)

8. The length and the width what?
   \( u₃ \ u₃ \ s₃ \ a₃ \ e₃ \ n₃ \ t₃ \ a₃ \ e₃ \ n₃ \ \)

9. 10° the surface pose
   \( 10° \ a₃-a₃ \ a₃ \ g₃ \ a₃ \ r₃ \ \)

10. and 9 (to) which he has doubled pose;
    \( 9 \ a₃ \ i₃₃ \ p₃ \ a₃ \ g₃ \ a₃ \ r₃ \ a₃ \ \)

\[ \sqrt{x} = 3 \]
\[ \{x = 3-(x-y)\} \]
\[ x = 3z \]
\[ (x-y) = \frac{1}{3} \ x = x \cdot z \]

11. The equilateral of 9 (to) which he has
    doubled what? 3.
    \( b-a₃, 9 \ a₃ \ i₃₃ \ p₃ \ e₃ \ n₃ \ t₃ \ a₃ \ e₃ \ n₃ \ \)

12. 3 to the length pose.
    \( 3 \ a₃-n₃₃ \ u₃₃ \ g₃ \ a₃ \ r₃ \ \)

13. 3 to the width pose.
    \( 3 \ a₃-n₃₃ \ s₃ \ e₃ \ n₃ \ \)

14. Since "so much as the length over the
    width goes beyond
    \( a₃ \-₃ \-₃ \ u₃ \-₃ \ u₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \ a₃ \-₃ \ e₃ \-₃ \ t₃ \-₃ \ u₃ \-₃ \ \)

15. I have made spon", he has sold
    \( u₃ \-₃-th₃ \ a₃ \-₃ \-₃ \-₃ \-₃ \ a₃ \-₃ \-₃ \-₃ \-₃ \ \)

16. 1 from 3 which to the width you have posed
    \( 1 \ i₃ \-₃ \-₃ \ a₃ \-₃ \-₃ \-₃ \ e₃ \-₃ \ a₃ \-₃ \ a₃ \-₃ \ k₃ \-₃ \-₃ \-₃ \ u₃ \-₃ \ \)

17. tear out: 2 you leave.
    \( u₃ \-₃ \-₃ \-₃ \ a₃ \-₃ \-₃ \ a₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \ \)

18. 2 which you have left to the width pose.
    \( a₃ \-₃ \-₃ \-₃ \-₃ \ a₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \ \)

19. 3 which to the length you have posed
    \( 3 \ a₃ \-₃ \ a₃ \-₃ \ u₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \-₃ \ \)

20. to 2 which to the width you have posed
    \( a₃ \ a₃ \-₃ \ a₃ \-₃ \-₃ \-₃ \-₃ \ a₃ \-₃ \-₃ \-₃ \-₃ \ \)
21. The 1 3 6 of 6 detach: 10'.
   1 3 6 pu-tur-na 10

22. 10' to 10' the surface raise, 1°40'.
   10 a-na 10 a-sa ša l l 1,40

   šu-ṣi, 1,40 en-nam 10

Obverse II

x = 3z = 3 · 10 = 30

1. 10 to 3 which to the length you have posed
   10 a-na 3 ša a-na uḫ ta-ša₂-ku-nu

2. raise, 30 the length.
   l l 30 ū[š]

y = 2z = 2 · 10 = 20

3. 10 to 2 which to the width you have posed
   10 a-na 2 ša a-na sag ta-ša₂-ku-nu

4. raise, 20 the width.
   l l 20 sag

Proof:

5. If 30 the length, 20 the width
   sum-na 30 uḫ 20 sag

6. the surface what?
   a-sa₂ en-nam

xy = 30 · 20 = 10'

7. 30 the length to 20 the width raise, 1°
   the surface.
   30 uḫ sum-na 20 sag l 11 10 a-sa₂

x² = 30 · 30 = 15'

8. 30 the length together with 30 make spon: 15°
   30 uḪ it-ti 30 bu-ta₂-kī₂-ša₂

x - y = 30 - 20 = 10

9. 30 the length over 20 the width what
goes beyond? 10 it goes beyond.
   30 uḪ a-gū 20 sag mi-nam li-tir 10 li-tir

(y - x)² = 10 · 10 = 1°40'

10. 10 together with 10 make spon: 1°40'.
   l l ti-ti (to ku-ta₂-kī₂-ša₂) 1,40

9 · (x - y)² = 9 · 1°40'

= 15°

11. 1°40' to 9 double: 15° the surface.
   1,40 a-na 9 a-gī₂-nī₂-ma 15 a-sa₂

x² = 9 · (x - y)²

12. 15° the surface is as much as 15° the
    surface which the length
    15 a-sa₂ ki-ni 15 a-sa₂ ša uḪ
13. by itself has been made spm.

Taken by itself, the phrase "šš uš ina ramānītu utikītu" could perhaps also be interpreted as "which I made the length spm by itself". The preposition ina occurs, however, in connexion with Kutikum in all four occurrences of the relative clause in question and nowhere else in the tablet (nor anywhere else, as far as I can find out). Elsewhere in the tablet Kutikum stands with ū, ṝana and ītī. The probability that this distribution should come about randomly is extremely small \(2.3 \times 10^{-6}\) in a reasonable stochastic model. Furthermore, the occurrences in obv. II,12f and rev. 2f stand in passages where the context requires the second person singular (because imperatives are pointed at) if the subject of the clause is not ūšence, the form cannot by the usual St(i)(causative, reflexive), but must be St(i)(passive of causative), of which this preterite form coincides with that of St(ii).

The choice of "he" instead of "y" as the subject of the doubling is enforced by related passages in VAT 8520, obv. 7, 9, 11, rev. 8, 10.

As usually, the main lines of the procedure can be mapped by the arithmetical representation. On a number of points, however, it is inadequate: Why is a width equal to the length of 3 introduced in 1,13 (if this is at all the meaning of the expression "pose to")? Which principles govern the use of the three multiplicative terms ("making spm"; "raising"; and "doubling to p")? Why are so many different entities spoken of as "surface"? Normally, such words stand as epithets which serve to identify a number; this is also the case in 1,22, where "10" the surface" is kept apart from ūš (the i of 61") -- but this function can only be hindered when \(x^2\) and \(9 \cdot (x-y)^2\) are also labeled "surfaces" (1,2f, II, 11f, 112). So, in some sense or other, all these entities must be "surfaces".

Further: Why are the "surfaces" "built", while other complex expressions are not (113)? And why are "posing" (e.g. "posing 10" the surface", in 1,9) and "posing to" (e.g. "posing 3 to the length", \(x^2\) and \(9 \cdot (x-y)^2\))
in 1.12] carefully distinguished all the way through the tablet? All these finer points of the formulation make no sense in the arithmetical interpretation; several appear to call for a geometric reading, and indeed, a geometric representation answers all the questions, while at the same time giving us some supplementary insight in the relation between "raising" and "making span".

The geometric representation which appears to be described in the text is shown in Figure 10, the relation of which to the 16 squares of BM 15285 No 10 is obvious. The "doubling to 9" of the square on the excess of length over width is clearly seen to be a concrete repetition, no multiplicative calculation. A width related to the number 3, and another width similarly related to 2, are clearly seen on the figure. And of course, all the "surfaces" are indeed surfaces in the most literal sense.

We observe that the numbers which are "posed" in 1.9-10 are "real values" — the real surface of the rectangle, and the number of repetitions of the small square. The numbers which are "posed to" length and width (in 1.12, 13 and 10), on the other hand, are not real values of the lengths and widths in question. It might seem as if "false values" (in the sense of a "false position") were "posed to" the entity for which they are assumed; still, according to normal Babylonian usage, later references (like that of 1.19) could then be expected to quote the assumed numbers as values ("3 the length which you have posed", or eventually "3 the false length which you have posed"). So, we are led towards the interpretation that "posing x to A" means "writing the number x along the entity A" — as it was suggested in Figure 9 (cf. note 108). Once again, the interpretation of BM 13901 No 10 as a subdivision rather than a comparison with an auxiliary figure is supported.
In one respect, the geometric interpretation changes the expectations which might be derived from the previous examples. When length and width, length together with length or excess together with excess give rise to rectangles or squares in I.1-5 they are "made span". So also in the proof, in II.8,10, when the length and the excess are squared. But in I.20, the number of small squares is calculated by "raising 3 to 2", and in II.7, "30 the length" is "raised to 20 the width". What is the difference? Are the terms synonymous in spite of all contrary evidence?

The clue has to do with the term "to build", and with the way triangular and trapezoidal areas are found. Only when a length and a width (or two other lines) have been "made span", is a surface said to have been "built". Conversely, when the area of a triangle, a trapezium or a trapezoid is calculated, the term used is invariably "raising". So, firstly, the terms cannot be synonymous. And, secondly, one of them must belong with the process of building and the other with calculation. In other word, the process "to make span" is to be a process of construction, and to "build" means "to construct" (in agreement with the Latin etymology of the latter word). "Raising", on the other hand, means "calculating by multiplication".

This agrees well with the use of the terms in our text. In the beginning, the rectangle, the square on the length and the square on the excess are all constructed now - none of them existed before. The number following the construction measures the area of the surface constructed -- so, the calculation of this area is implied by the construction process (140), but it remains something different. In I.20, when the numbers 2 and 3 are multiplied and the number of small squares in the rectangle thus calculated, the rectangle is already there; hence, 3 is
"raised to" 2, they are not "made span". (Cf. also BM 13901, No 23: the wāṣitone-corner is already there, there is no need to construct it, nor is the wāṣitone "made span" -- see above, note 101).

In the proof, the rectangle is still supposed to be there. In 11.7, the length is "raised to" the width. The squares on length and excess, on the other hand, are "spanned". Since the same pattern repeats itself accurately in the second problem, this can hardly be an accident. So, the squares are not there to the same extent as the rectangle -- either because only the rectangle is drawn, while the others figures are only imagined (3 and 2 being "posed" successively to the same width?) - or because everything is imagined, but the rectangle is more familiar as the basic figure and therefore still present to the inner eye. In any case it is made plausible that no complete figure like that of figure 10 was really drawn. Part of the procedure, if not all of it, was performed as mental geometry.
VI. The question of drawings

At this point it seems natural to ask whether the Babylonians left any traces of drawings like those of figures 4 to 10. The answer is, if we confine ourselves to algebraic texts like those to which these figures belonged\(^{(115)}\), a clear no.

This might seem to present a problem to the geometrical hypothesis. Truly, much "geometric" manipulation can have been performed mentally (and part of it must have been so performed, it appears from the above). But skill in mental geometry can only be acquired through familiarity with materialized geometry. So, as its a geometric interpretation of Babylonian algebra implies/basis a physically palpable representation of this geometry.

On the other hand, drawings are also absent from the tablets in other cases where we can be sure that the argument presupposes a geometric figure. True enough, some real geometric problems are accompanied by a drawing. Still, this drawing is only an illustration of the statement of the problem, not of the procedure --even in cases where auxiliary lines or appended figures are supposed by the argument they are left out from the drawing\(^{(116)}\). Furthermore, when the verbal statement of a geometric problem appears to be sufficiently clear, even the sketch of the geometric situation is often dispensed with.

So, even in cases where we can be sure that drawings have been made, they are absent from the tablets. This raises the question, where else they can have been made? Which medium can be imagined where drawings would leave no archaeological traces?

Several possibilities are open. The Greek drawings made in the sand are, at least from the engravings concerning the death of Archimedes, part of general lore\(^{(117)}\). For Mesopotamia, too, the use of the sand of the school courtyard has been proposed -
Figure 11. The fields belonging to the district Sulqi-sib-kalam, as drawn on the tablet MIO 1107 (left) and as redrawn in correct proportions by Thureau-Dangin (right). After Thureau-Dangin 1897:13, 15.

The dotted lines to the right correspond to lines which are very lightly drawn in the original. This weak incision is used by the scribe to mark lines the lengths of which have been calculated, not measured in the terrain.
namely as the medium for models of cuneiform signs in the basic scribal education\(^{(118)}\). Still, another possibility suggested by the Greeks is perhaps more interesting: The dust abacus. As explained above (chapter 11), the Greek term ἅφας, “abacus”, is in all probability derived from the Semitic root ḫbq, “to fly away”, “light dust”. On that background it seems plausible that the Greeks have first met the abacus in the form of a dust-board, and that they have done so in the Western Semitic area\(^{(119)}\).

As cultural connections between Syria and Mesopotamia were numerous (even much of the metrological system was shared), use of the same device in Mesopotamia is at least a strong possibility.

Whatever the medium of drawings corresponding to the solution of the geometric problems may have been, it left no traces (at least no traces which have been discovered until now). So, we need not worry much because no drawings corresponding to the solution of algebraic problems have been excavated.

On the other hand, drawings have been excavated which show us something about the probable character of the geometric support for algebraic as well as geometric problem solution,- to wit the field plans. The autography of one of these, as well as a re-drawing in correct proportions\(^{(120)}\), will show us how (see Figure 11).

Fig. 11
p. 66a

The first feature of the plan to be observed is perhaps the subdivision into right triangles, right trapeziums and rectangles. Subdivisions are of course not easy to do without when a natural area has to be measured, but the plan shows us

- that right triangles and trapeziums were looked for, not any triangle and trapezium. In the latter case, a height would have to be measured; right figures, on the other hand, are fully
described by \textit{length} and \textit{width} (in the case of right trapeziums two widths, "upper" and "lower").

- that the right angles of the partial figures were clearly marked on the figure, while no care was taken to render other angles correctly \textsuperscript{121}.

- and that the Babylonians were perfectly aware of the possibility to use auxiliary lines which were calculated, not measured (the calculation shows also awareness of the imprecision raising during measurement, since the dimensions of the partial figures are calculated in two different ways and the average found - whence the two writing directions for the partial areas).

Another striking feature is the total lack of care for a faithful rendering of proportions. A line is, so it seems, described by the number written unto it (if it is a line of importance for the determination of "lengths" and "widths" of the partial figures). One and the same line on the figure can even have two different numbers written unto it - this is the case of the line delimiting the two triangles to the utmost left: the numbers alone tell us that two different lines in the terrain are meant.

This lack of care for correct proportions has some curious effects. At bottom of the plan, the hypotenuse of a right triangle continues directly as the skew side of a trapezium. In reality, the two lines are at an angle somewhat below 120°. To state things a bit sharply, the Babylonians did not make a drawing of the terrain in their field plans: They made a \textit{structural diagram}, showing relevant lines, stating their lengths by inscribed numbers, and indicating their mutual relation with respect to the intended area calculation by visually right angles between lengths and widths.
Figure 12. The quadrangle of YBC 4675. Above two possible configurations (only the lengths of the four sides are stated in the enunciation). To the right, the diagram found in the tablet (after the photograph of HCT, Plate 26) -- not drawn but impressed with the stylus.

Figure 13. Autography of BM 15285, obverse. From Gadd 1922:156. The problem translated and discussed in section V.6 belongs to the reverse of the same tablet.
Similar structural diagrams are also often made as a support for the verbal statement of geometrical problem texts. A glaring example of the difference between the real figure and the diagram interpreting the structure of the problem is YBC 4675(122) -- see Figure 12.

Naturally, this does not mean that the Babylonians were unable to make real geometrical drawings when they wanted to, or that they did not recognize a geometrical square -- cf. Figure 13, which shows the obverse of a famous tablet investigating a variety of geometrical forms. Still, the use of structural diagrams instead of drawings in our field plan and in the geometrical problem text suggests that the geometrical drawings or imaginations which possibly supported the solution of algebraic problems may very well have been of the diagram type.

The first step in the reduction of Figure 6, the redrawing in reduced vertical scale, need not have been performed in drawing. At the evidence of field plans etc. we may surmise that the Babylonians can have been able to imagine the left section of the unshaded part of the figure first as a rectangle and next as square, while the right section would in both steps be considered an appended "one third of the side". At the same time, they will have known that the changed conception of the whole figure would correspond to a reduced area: No longer 20 but 40°·20' = 13°20".

Before leaving the problem of "drawings" we should take note of the fact that geometrical configurations can be represented materially by other means than through lines traced on a soft or colour-receiving surface. Some details of the Babylonian formulations could be read as hinting at a representation through small sticks or pieces of reed -- I think especially of the iden-
tification of rectangular figures and their sides and of the bissection through "breaking". It is also possible to make a pebble-representation of geometric figures in Greek style and to perform naive-geometric "algebraic" argumentation on such figures - and there exists indeed some evidence that early Greek calculators did so, inspiring thereby the development of the theory of figurate numbers\(^{(123)}\).

So, even though lines traced in sand or dust appear to be the most plausible candidates for a representation of naive-geometric algebra it should be remembered that they are not the only possible candidates.
VII. The first degree

All texts discussed up to this point were of the "second degree", if we translate them into modern formalism, and such problems are the main concern of the whole investigation. To a large extent, however, Babylonian mathematics dealt with real-life problems, which in the Babylonian context were of the first degree; furthermore, the more complex second-degree-problems involve transformations and equations of the first degree. Both in order to locate the use of naive-geometric methods correctly in relation to the complete structure of Babylonian mathematics and in order to grasp the methods of the complex second-degree-problems it is therefore of importance to get some idea of the techniques and ways of thought of Babylonian first-degree mathematics.

The present chapter presents two groups of texts suited for that purpose. Firstly I present two procedure-texts stemming from a still built larger group of problems/on the same concrete data; they are sufficiently complex to admit of some insight into the patterns of thought employed. Secondly come two texts (stemming from a single tablet) reporting a didactical explanation of the transformations of a first-degree-"equation".

On the basis of the insights gained from these texts it will be possible to proceed to further second-degree-problems involving supplementary first-degree-transformations, which will give us a more complete picture of the relations between first- and second-degree-techniques.
The problem deals with a domain composed of two partial fields amounting of areas $S_1$ and $S_2$. The first field yields a rent in kind to $r_1 = 4$ gur of grain per bur, while the second yields $r_2 = 3$ gur per bur (120). In the present problem, the total area is given to be $S_1 + S_2 = 30'$ (sar), while the difference between the total rents yielded by the two fields is given as $R_1 - R_2 = 0'20'$ (sila).

$1$ bur = $30'$ sar, $1$ gur = $5'$ sila.

Obverse I

$r_1 = 4$ gur/bur

1. From $1$ bur $4$ gur of grain I have collected.

$1$ na bür $k$am $4$ śe-gur $a$-$k$u-$u$m

$r_2 = 3$ gur/bur

2. From $1$ second bur $3$ gur of grain I have collected.

$1$ na bür $k$am $3$ ni-li(-in) $3$ śe-gur $a$-$k$u-$u$m

$R_1 - R_2 = 0'20'$ (sila)

3. The grain over the grain $8'20'$ goes beyond.

$3$-um $u$-gū $3$-im $8',20$ l-tir

$S_1 + S_2 = 30'$ (sar)

4. My meadows I have accumulated; 30'.

$g$-ar-im $g$-ar-m $3$-in

5. My meadows what?

$g$-ar-im $u$ a $a$-$n$-a$m$

The value of the practical unit bur is "posed" repeatedly in the learned unit sar, while the specific rents are posed directly, without the intermediate calculation, as

$r_1 = 4$ gur/bur

$R_1 = R_2 = 0'20'$ (sila/bur)

$r_2 = 3$ gur/bur

$S_1 + S_2 = 30'$ (sar)

6. $30'$ the bur pose. $20'$ the grain which he has collected pose.

$3$-ba-ra-an $g$-ar-rra $20$ a$a$-$n$-a$m $a$-$k$u-$u$m $g$-ar-ra

7. $30'$ the second bur pose.

$2$-ba-ra-an $a$a$-$n$-an $g$-ar-ra

8. $15'$ the grain which he has collected pose.

$i$-[i]$a$a$m $a$a$-$m $a$-$k$u-$u$m $g$-ar-ra
Similarly, $R_1$ and $S_1$ are "poised":

9. 8'20" which the grain over the grain goes beyond pose
8,20 š(a) še-um u-gü še-im i-ta-cu gar-ra

10. and 30' the accumulation of the surfaces of the meadows pose
ũ 30 ku-mur-ri a-āā gar-im-meš gar-ra-ma

The total surface $S_1 + S_2 = 30\' \text{(mar)}$ is bisected into two partial fields of 15' and 15', and the respective rents are calculated under the assumption that the original specific rents hold good for these two fields:

11. 30' the accumulation of the surfaces of the meadows
30 ku-mur-ri a-āā gar-im-meš

12. to two breaks: 15’.
ana ši-na bi-pl-me 15

13. 15' and 15’ until twice pose:
15 ū 15 a-di ši-ni-bu gar-ra-ma

First the specific rents are recalculated in units of sila/mar (expressed as "false grain").

Next the hypothetical total rents $R_2^*$ and $R_3^*$ are found through multiplication with the hypothetical areas of 15' Band:

14. The 1gi of 30’, the bur, detach: 2’.
1gi 30 bu-ri-im pu-tur-ma 2

15. 2’ to 20’, the grain which he has collected
2 ana 20 še še-im-ku-nū

16. raise, 40’, the false grain: to 15’ which until twice
1l 40 ša-um 1(lu) ana 15 š(a) a-di(l) ši-ni-bu

16a. you have posed
ka-aš-ku-nu

17. raise, 10’ may your head retain.
1 l 10 re-aš-ka (l)l(š)l-l

18. the 1gi of 30’, the second bur, detach: 2’.
1gi 30 bu-ri-im ša-ni-im pu-tur-ma 2

19. 2’ to 15’, the grain which he has collected
2 ana 15 še-im ša-im-ku-nū

20. raise, 30’, the false grain; to 15’ which until twice
1l 30 ša-um l1 ana 15 ša a-di ši-ni-bu

$R_2^* = 7'30'' \text{(sila)}$

20a. you have posed raise, 7’30’.
ta-aš-ku-nu 1 l 3,30
The difference between the hypothetical total rents is found:
\[ R_1 - R_2 = 10\,\text{'} - 7\,\text{'}30\,\text{"} = 2\,\text{'}30\,\text{"} \]
This difference falls 8\,\text{'}20\,\text{"} - 2\,\text{'}30\,\text{"} = 5\,\text{'}50\,\text{"} short of the real difference.

21. 10\,\text{'} which your head retains
   10 \( \text{Re\-esh-ka u-ka-lu} \)

22. over 7\,\text{'}30\,\text{"} what goes beyond? 2\,\text{'}30\,\text{"}
   it goes beyond.
   u-gu 7,30 mi-nu 1-te-r̃ 2,30 i-te-r̃

23. 2\,\text{'}30\,\text{"} which it goes beyond from 8\,\text{'}20\,\text{"}
   2,30 \( \text{Re-esh-ka i-te-r̃} \)

24. which the grain over the grain goes beyond
   u-gu 7,30 \( \text{Re-esh-ka i-te-r̃} \)

Obverse II
1. tear out: 5\,\text{'}50\,\text{"} you leave.
   u-sū-uk-na 5,50 te-zi-ib
2. 5\,\text{'}50\,\text{"} which you have left
   5,50 \( \text{Re-esh-ka} \)
3. may your head retain
   \( \text{Re-esh-ka i-te-r̃} \)

The increase of the difference between the total rents is found for a transfer of 1 out from the second to the first field; \( R_1 \) increases by 40\,\text{"}, \( R_2 \) decreases by 30\,\text{"}, and hence the difference increases by 40\,\text{"}-30\,\text{"}=10\,\text{"}.

The required total transfer is found through a division by 1\,\text{'}10\,\text{"} to be 5\,\text{"} (30\,\text{"}), which is then added to the first hypothetical partial field and subtracted from the second in order to yield the real "meadow":

4. 40\,\text{"}, the change, \text{I and 30\,"} (the change)
   40 ta-ki-lir-tam 30 (ta-ki-lir-tam)

5. accumulate; 1\,\text{'}10\,\text{"}. The iquam \text{I} know not.
   gur-gur-ma 110 i-gi-a(m) u-u-l i-de}

6. what to 1\,\text{'}10\,\text{"} I shall receive
   mi-nu a-na 1,10 lu-uš-kul-un
7. which 5\,\text{'}50\,\text{"} which your head retains gives me?
   5,50 \( \text{Re-esh-ka u-ka-lu i-na-di-nam} \)

8. 5\,\text{"} pose. 5\,\text{"} to 1\,\text{'}10\,\text{"} raise.
   5 gar-ra 5 a-na 1,10 ii
9. 5\,\text{'}50\,\text{"} will it give you
   5,50 it-ta-di-ilum
10. 5\,\text{"} which you have posed from 15\,\text{"} which until twice
    5 \( \text{Re-esh-ka u-ka-lu i-na-di-nam} \)
11. you have posed, from one tear out
    ta-ša-ku-nu i-na il-[a]-ta-en u-uš-uh
12. to the other append.
    a-na il-[a]-te-en il-[a]-lu-nu
13. The first is 20', the second is 10'.

\[
S_1 = 15' + 5' = 20' \text{ (not)} \\
S_2 = 15' - 5' = 10' \text{ (not)}
\]

Proof:
The total rents \( R_1 \) and \( R_2 \) are found for the values \( S_1 = 20' \) ar, \( S_2 = 10' \) sar (by renewed calculation of the "false grains")

14. 20' is the surface of the first meadow, 10' is the surface of the second meadow.

\[
20 \text{ a-\(\text{a}\) garim ë-te-\(\text{at}\) 10 a-\(\text{a}\) garim ë\(\text{a}\)-ni\(\text{ti}\)-\(\text{mi}\)}
\]

15. If 20' is the surface of the first meadow, 10' the surface of the second meadow, then the grains which he has collected:

\[
10 \text{ a-\(\text{a}\) garim ë\(\text{a}\)-ni\(\text{ti}\)-\(\text{mi}\) ë-\(\text{u}\)-\(\text{a}\)-\(\text{a}^{\text{-}}\) ennaa}
\]

16. 10' the surface of the second meadow, what? 10 a-\(\text{a}\) garim ë\(\text{a}\)-ni\(\text{ti}\)-\(\text{mi}\) ë-\(\text{u}\)-\(\text{a}\)-\(\text{a}^{\text{-}}\) ennaa

17. The igi of 30', the bur, detach: 2'.

\[
\text{igi 30 bu-ri-im pu-tur-ma} 2
\]

18. 2' to 20', the grain which he has collected:

\[
2 \text{ a-na 20 ë-ri-im ë-} \text{in-\(\text{k}^{\text{u}}\)-\(\text{u}\)}
\]

19. ro\(\text{i}\)se, 40'; to 20', the surface of the first meadow

\[
\text{igi 40 a-na 20 a-\(\text{a}\) garim ë-te-\(\text{at}\)}
\]

20. ro\(\text{i}\)se, 13'20' the grain, that of 20', the surface of the meadow.

\[
\text{igi 13,20 ë-um ë-} (20 a-\(\text{a}\) garim)
\]

21. The igi of 30', the second bur, detach: 2'.

\[
\text{igi 30 bu-ri-im ë-} \text{a-\(\text{a}\)-ni-\(\text{im}\) pu-tur-ma} 2
\]

22. 2' to 15', the grain which he has collected, ro\(\text{i}\)se, 30'.

\[
2 \text{ a-na 15 ë-ri-im ë-\(\text{in-\(\text{k}^{\text{u}}\)-} \text{u}\)} \text{ (igi) 30}
\]

23. 30' to 10', the surface of the second meadow,

\[
30 \text{ a-na 10 ë-} \text{a-\(\text{a}\) garim ë-\(\text{a}\)-ni\(\text{ti}\)-\(\text{mi}\)}
\]

24. ro\(\text{i}\)se, 5' the grain, that of 10', the surface of the second meadow.

\[
\text{igi 5 ë-\(\text{u}\)-\(\text{u}\)} \text{ (igi) ë-10 a-\(\text{a}\) garim ë-\(\text{a}\)-ni\(\text{ti}\)-\(\text{mi}\)}
\]

Finally, the difference between the rents of the two meadows is found to be 0'20' as required.

25. 13'20' \((\text{the grain of the first meadow})\)

\[
13,20 \text{ ë-um (ë-\(\text{a-\(\text{a}\) garim ë-te-\(\text{at}\)})}
\]

26. over 5 the grain \((\text{of the second meadow})\)

\[
u-\(\text{u}\)-\(\text{g}^{\text{u}}\) (5 ë-\(\text{a}\)-\(\text{a}\) garim ë-\(\text{a}\)-ni\(\text{ti}\)-\(\text{mi}\)}
\]

27. what goes beyond? 0'20' it goes beyond.

\[
\text{mi-nam i-tir} (8,20 i-tir)
\]
"seemum" translates gurum (-limurum), "(feld-)flur, Umrand, Umgebung". This name for a specific sort of field is possibly used because the normal name for a field (edum) is occupied in mathematical contexts as "surface". (The same word is used for partial fields in VAT 6512, cf. von Soden 1939:148, and covered by the logogram A-ENUG, perhaps in many other texts, cf. Thureau-Dangin 1940a: 47).

The plural of the "fieldes" is indicated by the suffix -med, which in the living Sumerian language had been replaced by a plurality of ummum (cf. Falkenstein 1931:37). Obviously, the Sumerograms of the text are abbreviations for Akkadian thought, and not evidence of an unbroken Sumerian mathematical tradition. Cf. also BN, 63, § 76.

"Gurum" is in the nominative form, Se'um. So, for once we are allowed by this happy opposition to interpret the common construction where a single number stands both as the result of one operation and as the object of the next: In the present case at least, the number is made explicit as a result, and is then implicitly understood in the next phrase.

This observation makes sense of a peculiar usage of the tablet BM 13901, viz. the use of the Sumerian agitative suffix -es as a separation sign between numbers. Indeed, Neugebauer made this explicit in his translation (e.g. in M 1, Obv. 1.1, translating the passage "at-me-er-ma 65-1 ka-si-cam" as "habe ich addiert und ge5 ist es. 1, den Koeffizienten"). Since the suffix is only used when a separation of a result from a succeeding number is required, I choose to regard the main function of the sign as a separation indicator, and absorbed it into the interpunctuation of the translation. There is, however, little doubt that a secondary agitative connotation is also implied by the sign.

"Change" translates takkitum, my conjectural restitution of the damaged words of the line. Both Neugebauer and Thureau-Dangin suggest ta-kk-il-tam, because this word was known to them as a mathematical term, which seemed to make sense, since they interpreted datakulum simply as multiplication and takkilum hence as a "factor". The profounder understanding of the term makes this reading meaningless and hence problematic. The only other word listed in ANW which seems to fit the remaining signs of the line is takkitum, "änderung". It is absent from other mathematical texts, but it turns out to make excellent sense in connection with an argument for which parallels are just as absent from our text material. Indeed, the term derives from the D-stem of našrum, viz. naškurum, "(ver)ändern", "bessern", "verbessern", "verbessern bringen", etc.

The restitution is only conjectural. Truly, Aage Westenholz finds it to fit the photograph at least as well as the old reading; but another trained eye, viz. that of von Soden, rejects it as impossible (personal communications).

The text appears to distinguish the iₚ, i.e. the reciprocal of a number (an abstract mathematical concept), from the table value jum, a very manifest entity. The latter term, in fact, turns up when the absence of the value from the table of reciprocals is stated. So does even the following text. Cf. YBC 6567, above, section V.1, which deals precisely with table values.
The double bracket [...] is used for a restitution of a passage where no parallel passages indicate the precise words of the original.

The mathematical commentary aligned with the translation shows that all steps of the procedure can be interpreted very concretely. In principle, the text can of course also be followed by an abstract symbolic calculation (in the way its correctness is proved in NK1). But the text contains many steps which are superfluous if we suppose the real procedure to have been abstractly algebraic or arithmetical -- so for instance the recalculation of the specific rents per sar in each case separately. The very complexity of the procedure points in the same direction: Why should the system

\[ s_1 + s_2 = 30' \quad s_3 (45_1 - 3s_1) = 8'20' \]

be solved via calculation of the quantity \[ s_1 - \frac{s_1 + s_2}{2} \]? In the text discussed immediately below, a still more spectacular detour (as viewed from the standpoint of abstract algebra) will turn up; finally, all problems from the group to which the present as well as the following text belongs can be followed in detail on the level of concrete thought. Even before we take the possible use of the term taklitum into account there seems to be little doubt that the real procedure is close to the one exhibited in the marginal commentary; if a collation confirms the possibility of the new reading, we can presumably regard the interpretation as fully confirmed, since no other replacement of the impossible taklitum seems at hand.

If we accept this conclusion, a number of features can be observed in the text. We observe that all intermediate quantities can be given a concrete meaning, either directly or, more significantly, with regard to a hypothetical situation. The "false grain" can be understood as "false" if we see it as that amount of grain which could be collected from the field in question had it been of area 1 sar; and the 2'30' [sila] of
obv. 1, 22 can be interpreted as the difference in rents had the
two fields been of equal magnitude.

The problem is of a type which in the Islamic Middle Ages
might have been solved by a "double false position". The
present text avoids the technicalization inherent in this proce-
dure and sticks to steps which can be intuitively and directly
justified. The text keeps far from understanding via abstract
arithmetical relationships; but it keeps equally far from the
use of schemes learned by heart, and close to procedures which
can be understood and explained.

Among the details of the text, the treatment of the bur de-
in obv. I, 17-21, serves closer attention. Both in the beginning of the text, and
again in the proof, "the bur" and "the second bur" are distinguished.
It appears that the value of the bur is not just taken note of
as a number when it is "posed" in the beginning: It is written
down or represented in two different calculation schemes or
concrete representations of the two fields.

We may compare this use of "posing" with that of obv. II, 6-9,
the division of 5'50' by 1'10'. The double construction of line
8 shows that "posing" is different from the arithmetical pro-
time of multiplication, the "raising", but at the same part of or
presupposition for the performance of the computation -- apparently
the "posing" in question stands for the insertion into a computa-
tional scheme or other fixed procedure.

A third function of the term is found in obv. I, 9-10: When
$S_1 = S_2$ and $R_1 = R_2$ are posed, it can have nothing to do with fixed
procedures -- the entities $S_1 S_2$ and $R_1 R_2$ are dealt with differ-
ently in the set of related problems. Apparently, these basic
given quantities are taken note of, presumably in writing, in
any case by some material means. We may assume that the way it is done is similar to the way burs and divisors are "posed" in computational schemes or fixed representations.

Our guarantee that "posing" of a given quantity uses some material means is provided by obv. 1.17 and 11.3. In both places, intermediate results are to be "kept in mind"—literally to be "held by the head". This is an expression which is only used for intermediate results, never when given quantities or quantities found by naive-geometric manipulations are taken note of. "Keeping-in-mind" appears to concern the recording of intermediate results which fall outside fixed procedures and computational schemes.

---


The two tablets VAT 8309 and VAT 8391 belong together, and contain a number of problems dealing with the same two fields. In the present problem, $S_1 - S_2$ and $R_1 + R_2$ are given (together with the values of the specific rents, which are common to all problems).

**Reverse 1**

<table>
<thead>
<tr>
<th>Given are again</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1 = 6$ gur/bur, and $r_2 = 3$ gur/bur</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Further</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1 - S_2 = 10^{10}$ sar</td>
</tr>
</tbody>
</table>

| 3. If from 1 bur of surface 4 gur of grain I have collected, |
| $\text{sumer} + \text{na} \ b\text{ùr}^{10} \ a^{-(4)} \ \#\text{e-gur} \ \text{am-ku-us}$ |

| 4. from 1 bur of surface 3 gur of grain I have collected, |
| $\text{in-a} \ \text{bùr}^{10} \ a^{-(4)} \ \#\text{e-gur} \ \text{am-ku-us}$ |

| 5. now 2 meadows. Meadow over meadow 10 yars beyond, |
| $\text{in-na-an-nu} \ 2 \ \text{gârim} \ \text{gârim} \ \text{u-gù} \ \text{gârim} \ 10 \ \text{i-tir}$ |
6. Their grain I have accumulated: 18°20'.

7. My meadow what?

garim-s-sa

8. 30° the bur pose, 20° the grain which
     he has collected pose.

9. 30° the second bur pose, 15° the grain which
     he has collected.

9a. pose.

gar-rā

Sā-Sū is "posed"

(though entity will be
designated S' in
the following)

10. 10° which meadow over meadow
goes beyond pose.

11a. 10  lā garim u-gū garim l-te-rū gar-rā

11b. 10  lā garim u-gū garim l-te-rū gar-rā

The wayin is "posed"

12. 1 the wayin pose.

13. 1 the wayin pose.

13a. 1 wayin-am pose

13b. 1 wayin-am pose

The specific rent of
the first meadow in
recollected in
sūl/gar

14. rauš, 40°; the false grain; to 10° which
     meadow over meadow goes beyond

15. rauš, 6°40'; from 18°20', the accumulation
     of the grain

16. let out: 11°40° you leave.

17. 11°40° which you have left, may your head retain.

18. 1 the wayin to two break: 30°.

19a. 30° on 30° until twice pose

19b. 30° on 30° until twice pose.
The specific rents \( r_1 \) and \( r_2 \) are re-calculated (at a second time) in sila/mar. The rents of the two halves of the unit sar are found:

20. The \( 1g_1 \) of 30°, the bur, detach: 2° to 20°, the grain which he has collected

\[ \text{ig}_1 30 \text{ bu-ri-im pu-šur-na 2 a-na 20 še-im ša in-kú-su} \]

21. \( rs \), 40°, to 30° which until twice you have posed

\[ \$1 40 \text{ a-na 30 ša a-di di-ri-šu ta-sá-šu-nu} \]

22. \( rs \), 20°, may your head retain.

\[ \$1 20 \text{ ce-sá-ka li-li-li} \]

23. The \( 1g_1 \) of 30°, the second bur, detach: 2°.

\[ \text{ig}_1 30 \text{ bu-ri-im ša-ni-im pu-šur-na 2} \]

24. 2° to 15°, the grain which he has collected

\[ 2 \text{ a-na 15 še-im ša in-kú-su} \]

25. \( rs \), 30°, to the second 30° which you have posed \( rs \), 15°.

\[ \$1 30 \text{ a-na 30 ša-ni-šu-im ša ta-sá-šu-nu} \$1 15 \]

26. 15° and 20° which your head retains

\[ 15 \$1 20 \text{ ša re-sá-ša ša-ka-li} \]

Hence, the rent of the average unit sar is 35°. Since the total rent of the area \((S_2+S_1)\) can be taken to come from such average sars \((S_2+S_1)\), and since it is known to be \(31.40°\), \((S_2+S_1)\) can be found through division by 35° to be 20°.

27. accumulate: 35°; the igum I know not.

\[ \text{gar-gar-ša 35 igi-ša u-úli-ad} \]

28. What to 35° shall I pose

\[ \text{mi-nam a-na 35 luš-šu-un} \]

29. which 11° 40° which your head retains gives me?

\[ \$1 11,40 \text{ ša t-as-ša ša ka-li ša-na-di-nam} \]

30. 20° pose, 20° to 35° raise, 11° 40° will it give you.

\[ 20 \text{ gar-ra a-na} \$1 35 \$1 11,40 ša-ti-dí-kum \]

By error, this area 20° is not bisected, which would give \(S_1\) and \(S_2\). Instead, it is confused with the area of the first meadow (which is indeed known in advance to be 20°). \(S_2\) is then found through the subtraction of 10° = \(S_1-S_2\).

31. 20° which you have posed is the surface of the first meadow.

\[ 20 \text{ ša ta-sá-ša-šu a-ša ša-gar-im lás-te-at} \]

32. From 20° the surface of the meadow, 10° which meadow over meadow goes beyond

\[ \text{i-na} 20 \text{ a-ša ša-gar-im 10 ša ša-gar-im u-ga-ša ša-gar-im i-t(a)-ru} \]

33. tear out: 10° the surface you leave.

\[ šu-ga-šu-ša \$1 \text{ a-ša ša} \text{ ti-li-b} \]
Reverse II
1-9 (contains a proof of no specific interest)

* wēšum is closely related to the wēšīnum of BM 13901, NoB 1, 2, 3 and 25 (cf. above, chapter V).

The basic conclusions could be repeated here: Once more, all more complicated steps in the calculation are chosen such that their results can be given a concrete meaning (and as before, simple transformations like that of bur/gur to šila/bur are performed without commentary). This time, however, there is direct and undamaged textual evidence for the correctness of the concrete interpretation given in the marginal commentary (128). Firstly, of course, the 35' of rev. 1.27 must necessarily be the rent of an average sar; secondly, the rent of 20' which corresponds to the semi-sar belonging to the first field is calculated with reference to "the bur", while the 15' corresponding to the second field is calculated with explicit reference (in rev. 1.23) to "the second bur", which all the way through belongs with the second field. The 35' is clearly not the rent of an abstract average sar but that of a sar composed half from one and half from the other field.

This confronts us with a terminological problem: It appears that the bisection of rev. 1.18 does not affect an area but instead a width of 1. Indeed, the wēšum which is already posed in rev. 1.12, and which is later bisected, is nothing but the masculine form of the wēšīnum known from BM 13901, the width of 1 which transforms a length into an area of equal magnitude.

Evidently, the term is supposed by our author to refer to a familiar quantity. Like the bur, it is "posed" (in rev. 1.12) for use in the calculation without being mentioned before among the given quantities.
The most obvious assumption is that the term means the same thing here as in the quadratic equations. If it does, we are provided with a clear exposition of the conceptualization of the calculation. The unknown area \( s_1 \cdot s_2 + s + s \) must be thought of as a rectangle of length \( s \) and width \( 1 \). Half of it, of length \( s \) and width \( 1 \) belongs to the first field, and the other half, of equal length and width, belongs to the second field. The 35 should not then be thought of strictly as the rent of 1 average ear, but as the rent of 1 unit length (1 nindan) of the rectangle; similarly, the division of rev. II.28-30 does not give us directly the area \( s \), but instead the length \( s \) of the rectangle, and thereby implicitly its area.

The idea may seem strange to us. But a related conceptualization appears to lay behind the area unit \( \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{e} \text{"standard width" of 1 nindan which permits us to see an area calculation as an operation of proportionality or scaling, and which thus gives conceptual unity to all applications of the term "raising" -- cf. Figure 3 and the discussion of the term meaning of the term in section IV.3.}
The two preceding texts treated seemingly concrete (if surely not practical) problems of the first degree. The present texts are very different. They deal with the basic abstract length-width-representation, and they solve no problems; instead, they present us with a didactical discussion of transformations of simple "equations of the first degree". They have been excavated in Susa (late Old Babylonian epoch), and they belong to a type not known from Babylonia itself. Maybe the need to fix didactical explanations in writing have to do with the fact that the texts represent a cultural import, no continuous autochtonous tradition; simply maybe the Susa excavators have had good luck where those working on (or looting!) Babylonian sites have not.

Although the two texts are mutually independent, they are so close to each other that both translation are best given together, before the commentary.

Part A

\[ x + y - ky = 45 \]

1. The 4th of the width from the length and width to tear out, 45. You, 45
   [\text{t-\text{et sa\-g} \text{i-na}}] \text{u\-s } \text{g a\-g z} \text{i 4} \text{5 za-e 4} \text{5}

\[ a \cdot (- = -) = 3' \]

2. to 4 raise, 3' you see. 3', what is that: 4 and 1 pose.
   \text{[i-na 4 i-\text{si} 3 \text{ta}-\text{mar} 3 mi-nu bu-na 4 3 1 g\text{ar}}

\[ x + y = 50, \text{ by} = 5 \]

3. 50 and 5, to tear out, pose. 5 to 4 raise, 1 width, 20 to 4 raise
   [\text{t-\text{ai} 5 z} \text{i 'g\text{ar'} 5 e-na 4 i-\text{si} 1 sa\-g 20 e-na 4 l-li}]

\[ 4 \cdot 5 = (4 \cdot ky =) 1 \cdot y \]
4:20 ± 1:20 = 0:20
4:30 ± 2:8 = 5:1

1° 20' = 20 ± 4:20 = y

5° = 1°

2° + 3° = (4:3) x + 3:3 = 3°

4° = 15°

5° = 10 x (6:1 x) = 30 = 5 x

4° = 15° ± (5:1)

The coefficient to y is found by an argument of type "single false position" to be (x ± 4:3) / 3:4 = 5:3 ± 15°:3 ± 45°

The coefficient to x is 1 (from line 6)

The "width" y of the calculation is known to be 9 times the "true width" (of a figure?); hence y = 1° ± 20°, and 45°:y = 45°:20 ± 15°, which when subtracted from 30° = 20:1 leaves 30:1 ± x

5° = 1° 20' you see, 4 widths. 30 to 4 raise, 2° you see, 4 lengths. 20, 1 width to tear out,

1° 20' te-mar 4 ság 30 a-nä 4 li-li 2 ta-mar> 4 uš 20 1 ság zü

5° from 1° 20', 4 widths, tear out, 1° you see, 2°, lengths, and 1°, 3 widths, accumulate, 1° you see.

j-na 1:20 4 ság zü 1 1:0 3 ság ul pal 1:0 1:0 3 ta-mar

6° The length of 4 detach, 15° you see, 15° to 2°, lengths, raise, 30 you see, 30 the length

ig 4 pu-[ši] 15 ta-mar 15 a-na 2 uš 1:0 30 1:0 3 ta-mar> 30 uš

7° 15° to 1° raise, 15° the contribution of the width. 30 and 15 return.

15 a-na 1 li-li 11 na-na-at ság 30 uš 15 li-li

8° Since the 4th of the width to tear out, he has said,

aš-sum 4-at ság na-aš-bu qa-bu-kü t-na 4 1 zü 3 ta-mar

9° The length of 4 detach, 15° you see, 15° to 3 rise. 45° you see, 45° as much as (there is) of widths.

ig 4 pu-[ši]-dr 15 ta-mar 15 a-na 3 li-li 45 ta-mar> 45 ki-nä (ság)

10° 1 as much as of lengths pose. 20 the true° width take. 20 to 1 rise, 20 you see.

1 ki-nä uš gar 20 gi-na ság le-gö 20 a-na 1 li-li 20 ta-mar

11° 20 to 45° rise, 15° you see. 15 from 30°

tear out,

20 a-na 45 li-li 15 ta-mar 15 a-na 30 15 (zü)

12° 30 you see, 30 the length.

30 ta-mar 30 uš

* THS transcribes the beginning of this line as "[SO 0] 5 Z.I.A(1) <GAR>" and interprets ZI as a (phonetically motivated) writing error for S1, which would give the
passage the meaning "50 and 5 which go beyond <pose>". The supposed AE is, however, damaged and clearly separated from the ZI. As far as I can see from the autograph, the traces might as well represent the lacking EA, which would give the reading "(50 g) 5 zi gas", "50 and 5, to tear out, pose". This has the clear advantage over the reading of NIS to be in agreement with the zi, "to tear out", of line 4, as well as with those of lines 1, 5 and 6. The latter of these, which is an explicit quotation of line 1, is written in syllabic Akadian, excluding any error. It is also this quotation which shows that the zi is thought of as an infinitive, not as a finite form - cf. below, note 6.

† "Contribution" translates mandaum, an abstract noun derived from manda, "to count". Etymologically, the meaning would be "the counting"/"the counting". However, the term is found only here and in two other Suse texts (NIS XII and XXIV). In one of these, its use is unclear, in the other the term is isolated by a break. Alt suggests hypothetically an identification with Hebrew and Aramaic a'nees, which in NARe (pp. 428-430) is exemplified by "Anteil der Priestler und Leviten" and "d. Teil ( Beitrag) des Königs". The ensuing "share/contribution of the widths" fits the present text excellently (and it is not contradicted by the other two occurrences).

‡ "Retain" is a conjecture (kili!) due to von Soden (1964:49). His has hubil, Assyrian for "way", interpreted as "method" by the editors.

§ This quotation is remarkable, since the ideographic zi is rendered syllabically by an indubitable infinitive, na-za-ju.

¶ This claims that an indubitable gi-na, "tree", must be a writing error for ki-ma, "as much as". If this were the case, ki-ma sagg, "as much as of widths", would represent both the coefficient to the width (45° in line 9) and the value of the width (20, in line 10).

Part B

13. The 4th of the width to that which length over width goes beyond append 1-te-re day

14. 15. You, 15 to 4 raise, I you see, what is that? 15 za-e 15 a-na 4 1-si 1 ta-mar mi-nu-bi-ru

15. 4 and 1 pose. (...) 4 1 gas (15 a-na 4 1-si 1 ta-mar mi-(nu-bi-ru)}
* "To scatter" translates μαθέωμα, "auflösen, zerstreuen" (the reading is due to von Soden - private communication, cf. 1964:49). In fact, 15 is "scattered", i.e. analyzed into its constituent components 10(ις-ν) and 5(αὐτά).

Written ri-ba-ti, a genitive plural. Cf. BM 13901, No 13, note (8).

† JHS reads "z 31-24" and neglects the "4" in the translation, since this number gives no sense. Often GAR (=gar, "to pass") and 4 cannot be distinguished; so, we seem to be left with the choice between a formulation which makes no sense in its context, but which could have crept in by a copying error (the reading of JHS), and a reading which makes sense, and which possesses an parallels in line 17 (the present reading). However, close inspection of the autography shows an outspoken tendency to write GAR symmetrically, while 4 is normally written asymmetrically (as Ψ and Π, respectively). Only collation could decide whether the few exceptions are due to the scribe or the copying, and whether the difference reflects a different sequence of impression of the wedges. In any case, the problematic sign is as much a GAR as its left neighbour. So, the reading "gar zi-ιγ" appears to be established beyond reasonable doubt. Cf. also part A, line 4

** JHS makes a different restitution, which presupposes that lānum, "to take", is used synonymously with رسوم, "to raise" as a term for multiplication. This assumption is totally unsupported, and clearly contradicted by part A, line 10.

The present restitution is conjectural - only the "raise" required by the "to" seems secure. Possibly the restitution fills out the entire lacunae, possibly a few more signs can have found their place.

Both parts deal with a length of 30 and a width of 20, and this is supposed by the text to be known in advance (133), as are the sum of length and width, the excess of length over width, and the fourth of the width.

Part A leads off with an equation which translated into symbols runs \(xy-\sqrt{y}=45\) and asks for the meaning of the \(y\) which result when the right-hand-side is multiplied by 4. It then looks at
the single members of the left-hand side, multiplying each with 4, explaining 4·20 = 1'20' to be 4y, 4·30 = 2' to be 4x, and 4·2 (subtractive 3) = 20 to be a subtractive width (cf. below on this indication of sign). The result is 2'8/(1'20'−20') = (the required) 3'.

Then, from line 6 onwards, the reverse operation is performed, but this time on the sum of 2' = 4x and 1' = 2y. 4·2' = 30 is told to be simply x, while 4·1' = 15 is told to be the "contribution of y". In line 8-9, the coefficient of y is calculated to be 4·1/(4−1) = 45', and it is given the name "as much as" (kiša) (there is) of widths. In line 10, the coefficient of x is stated to be 1. Finally, the product of y and its coefficient is calculated and subtracted from the 45 of the right hand side (written already as analysed in lines 6-7), and the remainder is seen to be equal to the length, as required.

Part B runs along similar lines, the main difference being perhaps that this time the analysis of the right hand side appears to be made verbally explicit as a "scatterino" in line 16. "Contributions" and "coefficients" recur - the former, it is true, without the explicit label manšūtu.

For the sake of clarity, the operations can be organized schematically, as it is shown on the following page.34a. We observe that there is a close analogy between the Babylonian text and our own treatment of the corresponding equation. Not only the coefficients and the contributions but also the multipliers 4 and 6 of the left margin are stated explicitly. It seems, however, that most of the operations are supposed to be followed mentally: only the multipliers and the numbers 50
Schematization of the operations of part A

\[
\begin{array}{c|cccc}
\alpha & 1 \times + \frac{1}{4} y & = & 45 \\
\beta & 1 \times + \frac{45}{y} & = & 45 \\
\gamma & 30 \times + 20 & - & 5 & = & 45 \\
\delta & 50 & - & 5 & = & 45 \\
\delta' & 30 & + & 15 & = & 45 \\
\epsilon & 4 \times + \frac{4}{y} & - & 1 & = & 3 \\
\zeta & 4 \times + \frac{3}{y} & = & 3 \\
\eta & 2' \times + \frac{1'20'}{y} & - & 1 & = & 3 \\
\theta & 2' & + & 1' & = & 3 \\
\end{array}
\]

Apparently, the "1" and "4" posed in line 2 of the text are the factors written to the left of the two groups of equations. The text discusses the relations between the lines 3 to 8.

It is seen that \(a\) represents the original equation of “lengths” and “widths,” written symbolically, while \(\epsilon\) is obtained from this original equation through multiplication by 4. \(\gamma\) and \(\eta\) represent the same equations when the known values of length and width are inserted.

In the text, line 3 “poses” the 50 and 5 of \(\gamma\), representing 5 as “that which is torn out” (from 50). Next (line 4-5), the transformation of \(\gamma\) into \(\eta\) is explained term for term in order to solve the problem raised in line 2: which meaning to ascribe to the 3' which arise when the right-hand side of \(\alpha\) is multiplied by 4. This is done with reference to \(\epsilon, \zeta, \eta, \) and \(\theta\).

Line 6-7 explains the reverse transformation \(\eta\) to \(\gamma\), referring to \(\delta'\), where the respective contributions of lengths and widths are separated. Line 8-12, finally, explains \(\delta'\) in terms of \(\beta\) where the coefficients of \(x\) and \(y\), i.e., “as much as there is” of lengths and widths, are found and multiplied by the numerical value of these entities.
and 5 of line 8 are "posed", in a way which suggests written or other material representation; all the rest is done rhetorically.

In the previous texts the concrete pattern of thought was noticed. A similar observation can be made here, both on the terminology used for contributions and coefficients and for the way the coefficients are calculated. In both parts, the coefficient of \( y \) is found by an argument of type "single false position" and not through the arithmetically simpler but more abstract calculation \( 1-\frac{4}{5} = 1-0.8 = 0.2 \). (Similar patterns are found elsewhere in the material, e.g. in VAT 7532, rev. 6.7 -- MKT I,295).

Even if concrete, the designation of the coefficient by a special expression can be considered a formalization of the technique "accounting" which was discussed above (section V.6). Another formalization of something which was done currently with or without formalization is the designation of certain numbers or entities as "subtractive", "to tear out" (in lines 3, 4, 17 and 27), written by the sumerogram \( zi \). That we are really confronted with sort of sign is most clearly demonstrated by lines 4 to 5, where "20, 1 width", is firstly given the epithet "to tear out", and afterwards really torn out.

\( zi \) is not only used to indicate subtractiveness but also for the subtractive operation ("tearing-out") itself, e.g. in line 1, as it is indicated by the preposition "from" (\( in\)). It is an old issue whether such occurrences should be Akkadianized in transliterations. Thureau-Dangin did so without hesitation, regarding the sumerograms as pure logograms which were read by the scribes as grammatical Akkadian and which should hence be read so by us. He was so confident about this that he did not indicate the sumerogram parenthetically, as it is done in e.g. TMS. Neugebauer, on the other hand, claimed that the ideograms
functioned as mathematical operators, not as words belonging to
current language (see e.g. MCT I, viii). Line 8 of part A shows
that Neugebauer was at least partly right: The statement is
quoted, but the ideographic writing zi is rendered in phonetic
writing as an infinitive, na-sa hym (the text is written without
"mimation", the final m of nouns and nominal verbal forms which
was gradually dropped). At least the term zi must, at least in
the Susa school, have been regarded as an ideogram for an abstract
mathematical operation, not as a logogram to be provided by correct
grammatical pre- and suffixes when read.

Indications exist that the restrictions to zi and to the Susa
school are superfluous. Indeed, if i b - n i, were read mithartum
(as claimed by Thureau-Dangin), how are we to understand changes
in the ideographic expression following Sumerian homophonic pat-
terns (i b to i b, n i, to n i)? How are we to explain the use in
certain texts (among which IM 52301, see below, section X.1) of a
term bagû, evidently an Akkadianized pronunciation of ba - s i,?
What are we, finally, to do about the distinction between the
Akkadianization bagû, the table value, and igî, the abstract
reciprocal number? It appears that certain Sumerograms were (at
least in certain text-types, among which the compactly written
series texts must be reckoned) regarded as ideograms, that they
were sometimes read in Sumerian and sometimes Akkadianized without
proper inflection in person and tense?

A final observation on the text concerns part A, line 10-11. Both
the formulation and the actual calculation are conspicuous.
Why is the width spoken of as a "true width"? And why is 45' widths
calculated not as 20 raised to 45' but in two steps, the true
width being first raised to 1, and the result next raised to 45'?

The imminent analysis of the text provides us with no answers;
below we shall see how at least a suggestion can be found in the
texts BM 13901 No 14 and TMS IX (sections VIII.1 and VIII.3, respec-
tively),- a suggestion which appears to be confirmed in TMS XIX,
cf. below, note (176).
VIII. Combined second-degree problems

In chapter V, a number of simple second-degree problem texts were presented and discussed, and in chapter VII we had a look at some very concrete first-degree problems. Together, the two chapters might convey the impression that Babylonian mathematics was not only concrete in its cognitive orientation but also simple (not to say simplistic). In order to counteract at least in part this misleading impression the present chapter shall present a couple of texts which combine the first- and second-degree techniques in various ways, demonstrating a bit of the sophistication to which Babylonian algebra was able to rise while remaining concrete and "naive". The last section of the chapter presents another didactical Susa text, which builds the bridge from simple to more sophisticated second-degree algebra.

VIII.1. BH 13901, No. 16 (KKT III.1.31, cf. INB 5.5)

Several other problems from the same tablet were already presented above in Chapter V. The present problem contains yet another second-degree equation, this time in two variables connected through a simple first-degree equation. Through substitution and use of the accounting technique, the problem is reduced to that dealt with in section V.5 and solved by the same procedure.

Obverse II

44. The surfaces of my two squared lines
I have accumulated: 25' 25''.

a - a₂tti mi-it-bar-ti-na al-mur-ma (25,25

45. The squared line two-thirds of the squared line
and 5 m ndon

mi-it-bar-tum ši-ni-pe-at mi-it-bar-tim (2 5 m nda)n
46. 1 and $40'$ and 5 overgoing the 40' you inscribe, 
   $1 \& 40 \& 5$ (t-e-le-mu 410 t-e-la-pa-at)

47. 5 and 5 you make spon, 25 inside of 25'25' 
   you tear out: $\frac{5}{25} \frac{5}{25}$ (t-u-ul-ta-kal 25 l-lb-bl 25,25 t-a-na-sa-ab-ma)

Reversal 1

1. 25' you inscribe. 1 and 1 you make spon, 1. 
   40' and 40' you make spon, 
   (25 t-a-la-pa-at 1 \& 1 t-u-ul-ta-kal 1 40 \& 40 t-u-ul-ta-kal)

2. 26'40'" to 1 you append; 1'26'40'" to 25' 
   you raise; (26,40 a-na 1 t-u-qa-ab-ma 1,26,40 a-na 25 t-a-na-si-ma)

3. 36'6'40'" you inscribe. 5 to 40' you raise; 3'20' 
   (36,6,40 t-a-la-pa-at 5 a-na 40 t-a-na-si-ma 3,20)

4. and 3'20' you make spon, 11'6'40'" to 36'6'40'" 
   you append; (3,20 t-u-ul-ta-kal 11,6,40 a-na 3(6),6,40 t-u-qa-ab-ma)

5. 36'17'46'40'" makes 46'40'" equilateral. 
   3'20' which you have made spon 
   (36,17,46,40 e 46,40 b-a-si 3,20 a t-u-ul-ta-ti-il-tu)

6. inside of 46'40'" you tear out; 43'20' 
   you inscribe (l-lb-bl 46,40 t-a-na-sa-ab-ma 43,20 t-e-la-pa-at)

7. The l-gi of 1'26'40'" is not detached. What to 
   1'26'40'"
   l-gi 1,26,40 u-la t-pa-ta-ar n-nam a-na 1,2(6,40)

8. shall I pose which 43'20' gives me? 30 its 
   b-o-an-d-o 
   (t-u-ul-ku-um 30 43,20 l-ni-di-nam 30 b-a-an-da-du)

9. 30 to 1 you raise; 30 the first squared line. 
   (30 a-na 1 t-a-na-si-ma 30) m-i-it-bar-tum i-t-ti-a-at

10. 30 to 40' you raise; 20; and 5 you append; 
   (30 a-na 40 t-a-na-si-ma 20) 45 t-u-qa-ab-ma

11. the second squared line. 
   (25 m-i-it-bar-tum l-a-ni-tum)
This calls for various observations. On one hand the operations correspond precisely to those of a modern solution to the same problem, or to those of a Medieval rhetorical solution. The Babylonians were fully able to reduce the problem to a basic type as were the Islamic algebraists or their more recent descendants, in spite of their concrete and geometric way of thought. On the other hand, the concrete and geometric method is present all the way through, not only in the final reduction of the basic problem \( ax^2 + bx = c \) (rev. 1, 2-9). The squaring of \((40 \cdot x + 5)\) appears to be imagined geometrically (cf. Figure 14); \(40 \cdot 40\) and 5-5 are made by "spanning", while the coefficient \(5 \cdot 40\) (an operation of proportionality, replacing "5 squared lines" by "\((40 \cdot 5) \) squared lines") is performed as a "raising". Great care is taken to take
the factor $l$ into account and to square it (rev. 1,1 and 9); the reduction to basic type, finally, avoids the unnecessary step to find the total number of "squared lines", which anyhow would have to be bisected.

If we go a bit closer to the text, we notice that the problem basic is reduced to the type of BM 13901 No 3 (section V.5); but the unknown "squared line" of this reduced problem is not identical with the greater "squared line" of the problem. Instead, the two squared lines of the problem are $1$ times this unknown and $40'$ times the unknown plus $5$, respectively (this is why the symbolic translation in the left margin introduces a variable $z$). An analogous distinction between a "true width" and a "width" obtained through a multiplication by $l$ could be found in THS XVI A, line 10. In both cases, the distinction can be said to be a distinction between an original problem and its "basic representation". In the present case, as mostly when concrete entities are represented, the representing entities are not mentioned by any name; we can only see from the calculational steps that a specific basic problem is dealt with (here that of No 3 of the same tablet, cf. section V.5).
Like BM 13901, this tablet belongs to the earliest documented phase of Old Babylonian algebra. The first three sections deal with problems of essentially the same structure \((x+y=5, xy+3z+2y=A)\) and might have been solved slavishly by the same procedure; instead, however, No. 1 and 2 make use of the same principle but apply it differently, while No. 3 goes quite different ways. The three problems taken together thus constitute a fine demonstration of the flexibility of Babylonian algebraic procedures -- had Babylonian mathematicians been nothing but a collection of standardized recipes, everything on the tablet had looked differently.

No. 1 was also the first Babylonian algebraic text for which a geometrical explanation was given -- viz. by Kurt Vogel as early as 1939. Finally, the problems are interesting because of various details in the formulations. As these can all be demonstrated on No. 1-2, I restrict the translation to these two problems, and explain No. 3 only in symbolic and geometric interpretation.

\[ x \cdot y = (x+y) \cdot 3.3^\text{a} \]

\[ x \cdot y = 5, \quad xy+3z+2y = A \]

\( u^8 \text{sag} \delta = b \text{sag} \text{ab-ta-kI-li-ma} \)

\( a \cdot \delta^\text{baan} \text{ab-ni-1} \)

\( u^8 \text{sun} \text{ma-li-1} \text{u} = \text{li sag} \)

\( \text{i-te-ru-6} \)

\( a \cdot \text{nsu li-bi} a \cdot \delta^\text{baan} \text{gI-li-ma} \)
6. 3°3'. I turn back. Length and width
7. accumulated; 27. Length, width and

\[ x + y = 27 \]

\[ x + y = 27 \]

7. accumulated; 27. Length, width and

\[ \text{sur} \text{face} \]

\[ \text{what?} \]

\[ \text{gar-gar-ma} \]

\[ 27 \]

\[ \text{u} \]

\[ \text{qag} \]

\[ \text{a} \]

\[ \text{a} \]

\[ \text{a} \]

\[ \text{a} \]

8. You, by your making,

\[ \text{at-ta i-na e-pe-ti-kas} \]

9. 27, the things accumulated of length and

\[ \text{width} \]

\[ \text{ki-im-ra-te-su} \]

\[ 3 \]

\[ \text{a} \]

\[ \text{a} \]

\[ \text{a} \]

10. to the inside of 3°3'. append:

\[ \text{a-na li-bi} \]

\[ 3,3 \]

\[ \text{gi-lb-ma} \]

11. 3°30'. 2 to 27 append:

\[ 3,30 \]

\[ 2 \]

\[ \text{a-na} \]

\[ 27 \]

\[ \text{gi-lb-ma} \]

12. 29. HALF-PART of it, that of 29, you break:

\[ 29 \]

\[ \text{BA.A-} \]

\[ \text{BA.A} \]

\[ 29 \]

\[ \text{ta-be-ag-pe-e-ma} \]

13. 14°30' steps of 14°30', 3°30'15'.

\[ 14,30 \]

\[ \text{a-ré} \]

\[ 14,30 \]

\[ 3,30 \]

14. From the inside of 3°30'15'.

\[ \text{i-na li-bi} \]

\[ 3,30 \]

\[ 15 \]

15. 3°30' you tear out:

\[ 3,30 \]

\[ \text{ta-na-sa-ah-ma} \]

16. 15' the remainder. 15' makes 30'

\[ \text{equilateral} \]

\[ 15 \]

\[ \text{as-pi-ni-tun} \]

\[ 15 \]

\[ \text{e} \]

\[ 30 \]

\[ \text{e} \]

\[ \text{a} \]

\[ \text{a} \]

\[ \text{a} \]

17. 30' to the first 14°30'.

\[ 30 \]

\[ \text{a-nna} \]

\[ 14,30 \]

\[ \text{iš-te-en} \]

18. append; 15 the length.

\[ \text{gi-lb-ma} \]

\[ 15 \]

\[ \text{u} \]

\[ \text{u} \]
\[ Y = \frac{\sqrt{y^2 - x^2}}{2} \]

19. '30' from the second 14'30'
   30 ina 14,30 en-en-il

20. you cut off: 14 the width
   ta-ke-re-ni-3a 14 a-na

21. 2 which to 27 you have appended
   2 18 a-na 27 zu-an-3a

\[ y = y-2 = 12 \]

22. from 14, the width, you tear out:
   ina 14 sag ta-na-na-an-3a

23. 12 the true width.
   12 sag gi-na

Proof:

24. 15, the length, 12 the width, make span:
   15 u-3 12 sag u-3-te-kiki-3a

25. 15 steps of 12, 3' the surface.
   15 a-ra 12 3 a-3a

26. 15, the length, over 12, the width,
   15 u-3 u-ii 12 sag

\[ x-y = 3 \]

27. what goes beyond?
   mi-na wa-te-ar

\[ y + (x-y) = 3' + 3 = 3'3' \]

28. 3 it goes beyond; 3 to the inside of 3', the
   surface, append,
   3 it-te-ar 1-in-1 3 a-3a gi-3-

29. 3'3' the surface.
   3,3 a-3a

* Thureau-Dangin translated "length, width" (u-3 a-na) simply as "rectangle" (e.g., THB, 64). That this is indeed the correct interpretation of the composite expression is confirmed by the Susa table of constants (THS III, 32), which speaks of the "diagonal of length and width", meaning the diagonal of a standard rectangle of sides 45' and 1.

** This arrangement of the statement between lines 7 and 8 follows the autography (MKT 11, plate 35).
30. Length, width. Length and width
uš sa₇ uš ū sa₇

31. I have made span: A surface I have built.
u₆-ta₈-ki₇-li₇, na a₈₂₇₁₇ ab-nil

32. I turn around. The half of the length.
a₈₂₇₁₇ bi₇ mi₉-li₇ uš

33. and the third of the width
ǜ₆₈₂₇₁₇-u₇-ti ₇ sa₇

34. In the inside of my surface
₇₈₁₆ li₇ bi₇ a₈₂₇₁₇

35. I have appended: 15.
₃₆₇₁₆-li₇-na ₇

36. I turn back. Length and width.
₇₈₁₆-tu₇ uš ū sa₇

37. I have accumulated: 7.
₃₆₇₁₆-u₇-ur₇-na ₇

II

1. Length and width what?
s₇ ū sa₇ a₈₂₇₁₇₇

2. You, by your making,
₇₆₇₁₆-ta₁₇-na a₈₂₇₁₇₇-bi₇-

3. 2 (as) inscription of the half
₇₆₇₁₆-a₈₂₇₁₇-(a)-at₇ ti₇ mi₉-li₇-li₇-

4. and 3 (as) inscription
₇₆₇₁₆-na₇-at-

5. of the third you inscribe:
₇₆₇₁₆-u₇-ti₇ ta₈₂₇₁₇₇-pa₇-at₇-

6. The 11 of 2, 30°, you detach:
₇₆₇₁₆-li₇ ₂₇₁₆-₄₂₇₁₇₇-ta₈₂₇₁₇₇-pa₇-

₃₆₇₁₆₇-(xy) = 3°30'

7. 30° steps of 7, 3°30'; to (the place of) 7,
₃₆₇₁₆ ₇₈₁₆ ₃₆₇₁₆₇ a₈₂₇₁₇₇ ₇
8. (of) the things accumulated, length and width,
  \[ \text{ki-im-ra-tim u} \uparrow \text{ u saq} \]

9. I bring:
  \[ \text{ub-} \text{ba-si(l)-ma} \]

10. 3'30" from 15, my things accumulated
  \[ 3'30 \text{ i-na} \text{ 15 ki-[im]-ra-ti-[i]-a} \]

11. cut off:
  \[ \text{bu-ru-ug}-\text{ma} \]

12. 11'30" the remainder.
  \[ 11'30 \text{ \&a-pl-[i]-tu-m} \]

13. Go not beyond, 2 and 3 you make spon:
  \[ \text{a} \text{ la} \text{ wa-[i]a} \text{3} \text{ u} \text{3 u}-\text{ta-kal}-\text{ma} \]

14. 3 steps of 2, 6.
  \[ 3 \text{ a-râ 2 6} \]

15. The igi of 6, 10' it gives you.
  \[ \text{igi} \text{ 6 \text{ q}â} \text{10 i-na-dî-ki-mu} \]

16. 10' from 7, your things accumulated
  \[ 10 \text{ i-na} \text{ 7 ki-im-ra-ti-i} \text{-ka} \]

17. of length and width I tear out:
  \[ \text{u} \text{b} \text{ u saq a-na-}=\text{al}-\text{ma} \]

18. 6'50" the remainder.
  \[ 6'50 \text{ \&a-pl-[i]-tu-m} \]

19. HALF-PART of it, that of 6'50", I break:
  \[ \text{BA.A}-\text{a-[u]} \text{ qa} \text{6} \text{.50 e}-\text{pe}-\text{e}-\text{ma} \]

20. 3'25" it gives you.
  \[ 3'25 \text{ i-na} \text{-di-tu} \]

21. 3'25" until twice
  \[ 3'25 \text{ a-di} \text{-di-ni} \text{-nu} \]

22. you inscribe: 3'25' steps of 3'25',
  \[ \text{ta-la-pa-at} \text{-ma} \text{3} \text{.25 a-râ 3} \text{.25} \]

23. 11'40'25"; from the inside
  \[ 11'40'25' \text{ \text{a} [i] } 11'-\text{bl} \]
24. 11'30' I tear out
   11,30 a-na-sa-ab-aa

\[ x = \sqrt{10'25' \times 25'} \]

25. 10'25' the remainder. <10'25' makes 25' equilateral>
   10,25 ù-a-pi-il-tum <10'25' - 25' 6b = a16>

\[ x = \frac{xv + xy}{2} = \frac{3'25' + 25'}{2} = 3'50' \]

26. To the first 3'25'
   a-na 3,25 jà-te-en

\[ x = \lambda + 10' = \frac{3'50' + 10'}{4} \]

27. 25' you append: 3'50',
   25 tu-ga-an-na 3,50

\[ y = \frac{xv - xy}{2} = \frac{3'25' - 25'}{3} \]

28. and (that) which from the things accumulated of
   ù as j-na ki-im-ra-at

29. length and width I have torn out
   u6 ù sag a-zi-ga-gu-um

30. to 3'50' you append:
   a-na 3,50 tu-ga-an-na

\[ y = \frac{xv - xy}{2} = \frac{3'25' - 25'}{3} \]

31. 4 the length. From the second 3'25'
   4 u6 j-na 3,25 ù-a-ni-im

32. 25' I tear out: 3 the width.
   25 a-na-sa-ah-aa 3 saq

32b. the things accumulated
   7 ki-im-ra-tu-ù

32c. 4 length 12 surface
    3 width
    4 u6 12 a-6a

* Since kisratum is written in the status rectus (ki-im-ra-ti-g) and not in status constructus, "length and width" must stand (in this single case) as an apposition, not as the second member of a genitive construction. Hence the translation.

** In most of its occurrences, kisratum stands so that it cannot be decided whether a (most peculiar) singular feminine kisratum or a plural kisratum is meant. The inchoative plural of II, 39a could eventually be explained away (Thureau-Dangin, 194, 67, does so, translating "7 oct 15", i.e. "seven omens"). In II, 16, however, there can be no doubt that a single sum is spoken of in the plural, as ki-im(ri-ti-g). The ki-im(ri-ti-g) of II, 10 is also a most certain plural.

It is noteworthy that the singular form to be expected
Designating as usual the length as $x$ and the width as $y$, finally transcribe problem 3 as follows:

$$xy + (x-y)(x+y) = 1'' 13' 20''$$

and from the way the solution is formulated it is clear that the author was aware that this was equivalent to

$$x + y = 1' 40''$$

which could easily be reduced to a standard problem $xy = A$, $x + y = B$ by the method already known from Nos 1-2. Instead, however, the following steps occur:

$$(x+y)^3 = 2'' 46' 40''$$

$$(x+y)^3 - xy - (x+y)(x-y) = 1'' 23' 20''$$

which, putting $x + y = 1' 40'' = a$, reduces to

$$y' + ay = 1'' 33' 20''$$

whence

$$y' = 1'' 33' 20'' + (1' 40''/2)^2 = 2'' 15''$$

$$y = \frac{a}{2} = y + \frac{x+y}{2} = \sqrt{(1' 40'' - 1' 30'')} = 1' 30''$$

$$\frac{x-y}{2} = (x+y) - (y + \frac{x+y}{2}) = 1' 40'' - 1' 30'' = 10$$

and so finally

$$x = \frac{x+y}{2} + \frac{x-y}{2} = 50 + 10 = 60$$

$$y = \frac{x+y}{2} - \frac{x-y}{2} = 50 - 10 = 40$$

-- all of it formulated of course the usual way. The procedure is fully correct but it looks rather queer in the above symbolic transcription.

Let us first look at the procedures which appear to be
Figure 15. The geometrical interpretation of AO 8862 No 1. Distorted proportions.
used to solve the three problems. The steps of problem 1 can be
Fig. 15 easily followed on Figure 15. The simple addition of one length
and one width (regarded as rectangles of width \( r \), which is not
said explicitly) transforms the :regular surface of area \( 3 \cdot 3' \)
into a rectangle of which the area and the sum of length \( x \) and
width \( r \) are known. A bisection of this known length \( x+y = 29 \),
to which the rectangle \( x \cdot r \) is "applied with defect", allows the
reconstruction of the rectangular area as a gnomon. The area and
hence the side of the small square enclosed by this gnomon is
found, and the original dimensions of the rectangle \( x \cdot y \) follow
as usual. In this way, everything labelled "length", "width" or
"surface" is indeed a length, a width or a surface.

We observe that the procedure is different from the one shown
on Figures 4-d, which corresponded to "application with excess".
The corresponding problem in one variable is the type \( ax \cdot x = \beta \)
-- to give it a formulation which could be formulated inside the
Babylonian framework: "From a square lines I have torn out the
surface: \( \beta \). This is the type which has two positive solutions;
it seems to be completely absent from the Babylonian materi(137)
ev even though the problem in two variables is very common.

The reduction of \( N0^2 \) is somewhat more complex, but follows the
Fig. 16 same pattern, see Fig. 16. Figure 16A shows the configuration as
we would imagine the geometic situation described, while Figure
16B describes what appears to correspond more or less to the Baby-
lonian understanding, as described in the text. The numbers
"2" and "3" are "inscribed as inscriptions" of \( 1/4 \) and \( 1/6 \), probably
along the edges of the rectangle, to remind that the widths of
these edges are to be understood, not as \( f \) but as stated,- and when
\( \frac{1}{2} x + \frac{1}{2} y \) is to be subtracted from the aggregated surface it is
"brought to" the place of "length and width", viz. to those
Figure 16. The geometrical interpretation of AO 8862 NO 2. Distorted proportions.

Figure 17. The geometrical interpretation of AO 8862 NO 3.
entities which were accumulated — it is indeed clear from the text that the 3°30' is not brought to an abstract sum (which would also be mathematically meaningless) but to the collection of added entities (a point where the plural and hence concrete character of kirmānūm is of importance). The absence of a "turning"-clause between II.5 and II.6 shows that the inscription of 2 and 3 and the "bringing" are considered connected members of the same process — and thus that both regard the same figure.

When the half-sum of length and width is brought to the place of length and width, i.e. to the edges of the rectangle, it is obvious (and not commented upon) that the ½-length is eliminated; but more than ½-width goes away, and a curious calculation in II.13-15 finds the resulting defect to be 10' [width]. The process of "making 2 and 3 span" can be imagined as in the lower left corner of Figure 16A; but an independent procedure as shown in Figure 16C seems more plausible, among other things because of the explicit order to stop the ongoing procedure. In sort of parenthesis, an entity is "built" of which both ½ and ⅓ are easily taken, to allow for a two-dimensional variant of the "single false position" — cf. below.

From here on, everything runs as in No. 1.

Fig. 17 The geometrical reading of No. 5 is shown in Figure 17. It turns out that the squaring of x+y gives us a figure from which the given surface xy=(x-y)(x+y) can easily be torn out. The figure is seen to be of precisely the same structure as that shown in Figure 2, and other texts suggest that it was familiar in the Old Babylonian period too. What remains is a square of side y and a rectangle of sides y and x-y. This remainder is easily rearranged as a gnomon, as done in Figure 17B. The usual quadratic completion yields a side of the completed square equal to 1°30'.

If the rearrangement had been thought of as a problem in y (the saw), y²+50·y = 1°33'20", it might have been natural to subtract 50 from
this side y = 50 \times 1.30'$. Instead, however, it is subtracted from
the side of the square of Figure 17A -- and if we look at the
subdivision of this square through the quartering lines it is
indeed evident that the difference between the two entities
is the half-difference between the length and the width of the
original rectangle. It seems thus as if the steps shown in Figure
17B shall not be apprehended as a change of problem; instead, every-
thing is to be understood all the way through in terms of the
constituent parts of Figure 17A. By extension, we may surmise
that the "changes of variable" to $x$ and $X$ in Nos. 1 and 2 are not
really to be understood as explicit changes of the unknown;
that is indeed a comprehension inspired by rhetorical or symbolic
algebra where certain entities are distinguished by their own
name and hence regarded as fundamental unknowns. Instead, all
entities in a figure which are not known are unknown on an equal
footing as far as the solving procedure is concerned; only as
initially far as certain entities are asked for they can they be considered
privileged (and relatively privileged only, as the entities
asked for in the beginning and those found in the end need not
coincide). This corresponds to our own comprehension of
problems of geometrical analysis (the phrase to be understood in
its Greek sense).

A number of features of the texts call for separate discussion.
Most important among these is the occurrence of the term "râ",
"steps of", the multiplicative term of the multiplication tables.
In some places it stands alone, but time after time it is found
in double constructions that show the isolated occurrences to be
ellipses. Other texts state that a rectangle is to be built
from a length and a width, and leave the numerical multiplication
implicit, giving directly its result. In the present double
constructions, both steps are spelled out explicitly, the multi-
plication apparently through reference to the auxiliary tables --
and in 1.13 and in two places in No 3, it is the building process which is left implicit (41).

Another terminological peculiarity of the text is the use of the subtractive term harāqum, "to cut off", along with the more current nasāhum, "to tear out". Already from the metaphorical contents of the two terms we might expect that the latter would be preferred for identity-conserving subtraction from surfaces and the former for the shortening of one-dimensional entities, if a distinction were to be made. This is, indeed, precisely the main tendency of this as well as all other texts where the terms are found together; but it is only a tendency, in the sense that nasāhum may be used for one-dimensional entities too (most clearly this is seen in 1.19-22: first 30 is "cut off" from 14\"30\", and next 2 is "torn out" from the resulting 14) (42). It is thus excluded to regard the two terms as names for distinct operations. At the same time the tendential distinction prevents us from seeing the terms as connotationally neutral technical terms, whose metaphorical basis had been completely worn off. They constitute instances of mathematical terms which must be "regarded as open-ended expressions which in certain standardized situations are used in a standardized way" (as formulated above, note 29).

A third formulation of interest is the recurrent 8a.ā-šu ša, "half-part of it, that of", which is found in all three problems at the point where a rectangle is bisected in order to allow a gnomonic reorganization (I,12; II,19; III,13). The use of the determinative pronoun ša shows that the quantity pointed at, the one which is to be bisected, must have some independent existence, mental or physical, which allows us to think of or point at a definite entity. I.12, for instance, cannot be read as the bisection of an abstract number 29; it must by necessity deal with something definite -- another confirmation of the concreteness inherent in the naive-Babylonian interpretation.
terminological
A final point to be observed is the distinction which is main-
tained between mibbum, "half", and bārum, "half-part", and the
corresponding distinction between multiplication by l 1 - 2 - b i =
30" (NO 2, IV.6) and "breaking". Once more "breaking" is seen to
be reserved to describe bisection into natural "wings" — cf.
section IV.5, and note (m) to BM 13901 NO 1, section V.2.

As concerns the mathematical aspect of the texts, the flexible
handling of problems and methods was already pointed out in the
introductory remarks. It makes clear that the understanding behind
the text must have been flexible too, that it has nothing
to do with blind application of fixed rules or algorithms disco-
vered by equally blind luck.

Another/implication of the tablet concerns the purpose of such
texts. I think of the tabulation between 1.7 and 1.8. Here, before
the description of the solving procedure, the whole construction
and solution of problem 1 is told in advance. The subsequent
procedural prescriptions can therefore hardly be seen as an attempt
to find the unknown dimensions of the rectangle. The aim is not
really to solve the problem and find the solution; it is to
demonstrate how to solve the problem, to present an argued solu-
tion.

The calculation in NO 2, IV.13-15, finally, is remarkable,
though belonging more on the level of details. The Babylonian pre-
dilection for argumentation by means of a "single false position" was
pointed out repeatedly above (sections V.6 and especially VII.3). Here, however, the trick is extended into two dimensions, as revealed by the term "making span" (extension apart, its relation to the calculation of $1 \frac{1}{2} = 45'$ in TMS XVI is obvious). Since $\frac{1}{2}$ is stated directly to be $10'$, the identities $\frac{1}{2} = 30'$ and $\frac{1}{2} = 20'$ can hardly have been considered secret. The calculation of their difference through a geometrical subtlety must therefore be seen as a didactical nicety, as a means to demonstrate the extension of the simple argument.

VIII.3. TMS IX (TMS, 63ff., cf. von Boden, 1964)

Such didactical concerns are even more obvious in the Suse text TMS IX, which approaches the style of TMS XVI (above, section VII.3). In this case, however, the text goes from simplest ("$xy+x = 40'$") to less simple ("$xy+xy = 1'$") fundamental equation, ending with a fairly complex application of the fundamental principle.

Unfortunately, the transcription in TMS is not very precise, the restitution of damaged lines and the translation are worse, and the mathematical commentary is at times nonsensical. Had it not been for these circumstances, the text would probably have changed much conventional wisdom in the understanding of Babylonian mathematics 25 years ago.

**PART A**

1. The surface and length ACCUMULATED, $40'$ (30'-the length?) 20' the width)

   $x-y \cdot 1 \cdot x = 40'$

   Alternative approaches to an understanding:

   i-na-ma 1 u 5 -ma 10 (a-ša daš)

2. As length to 10', the surface has been appended
3. Either 

\[ \text{APPEND} \text{ to } 20', \text{ the width} \]

\[ \text{} \text{d-ul} \ 1 \ \text{KI.GUB.GUB a-na 20 [seg]} \]

or

\[ 1^20' - 20' = 40' \]

4. or 

\[ 1^20' \text{ to the width which 40' together} \]

\[ \text{WHICH THE LENGTH (SURROUNDS)} \]

\[ \text{} \text{d-ul} \ 1,20 a-na \ seg \ â\ 40 \ it-ti \ u\ (\text{NGIN}) \]

or

\[ 1^20' - 30' = 40' \]

5. or 

\[ 1^20' \text{ together with 30' the length} \]

\[ \text{MAKE SURROUND, 40' its name} \]

\[ \text{} \text{d-ul} \ 1,20 \ it-ti>30 \ u\ \text{NGIN} [\text{NGIN}] \text{40 SUM} \text{-[TIG]} \]

Implicit conclusion:

\[ x:y+1:x=x:(y+1) \]

6. Since so, \[ 20', \text{ the width, which} \]

\[ \text{he has sold to you} \]

\[ \text{a}\-\text{SUM} \text{KI-SAN a-na 20 seg} \ â\ \text{GA-BU-KU} \]

7. \[ \text{i appended;} 1^20'' \text{ you see. Out from here} \]

\[ 1 \ \text{dah-NA} \ 1,20 \ \text{TA-NA} \ \text{TA-NA a-na-ki-a-am} \]

8. \[ \text{you ask, 40' the surface, 1^20' the width,} \]

\[ \text{the length what?} \]

\[ \text{TA-NA} \ 40 \ a\-\text{SUM} \ 1,20 \ \text{seg} \ u\ \text{mi-du} \]

9. \[ \text{30 the length'}. \text{So the having-been-mode} \]

\[ \text{30 u\-} \text{KI-SAN na-PA-SUM} \]

\[ \text{---------------------------} \]

\[ \text{PART 8} \]

\[ \text{(Surface, length and width)} \]

\[ \text{AC'\text{CUMULATED, 1. By the Akkadian}} \]

\[ \text{} \text{a\-\text{SUM} u\-} \text{seg UL.GAR a-na a-na}\-\text{DI} \]

\[ \text{(x+1):(y+1)} \]

\[ = x:y+1:y+1:y+1:1 \]

11. \[ 1 \text{to the length append.} \text{1 to} \]

\[ \text{the width append. Since 1 to the length} \]

\[ \text{is appended,} \]

\[ (a-na u\-dah) a-na seg dah a\-\text{SUM} a-na \]

\[ u\-dah \]

12. \[ 1 \text{to the width is appended, 1 and} \]

\[ \text{MAKE SURROUND, 1 you see.} \]

\[ (a-na seg dah 1 ü 1 \text{NGIN} \text{1\-TA-NI} \text{TA-NA} \]

\[ (x+1):(y+1) = \]

\[ x:y+1:y+2 \]

13. \[ 1 \text{to the ACCUMULATION of length,} \text{width} \]

\[ \text{and surface append, 2 you see} \]

\[ (a-na UL.GAR u\-seg a\-\text{SUM} a-na \text{TA-NA}} \]

\[ Y:y+1:1^20' \]

14. \[ (1^20' \text{the width 1 append and} \] \[ 1'30' \]

\[ X:x+1:1^30' \]

\[ \text{To} \]

\[ \text{30' the length 1 append, 1'30'.} \]

\[ (a-na 20 seg 1 dah) 2,20 a-na 30 u\-dah 1\text{ dah}, 30 \]
15. (Since a surface) is, that of 1'20" the width, that of 1'30" the length
(la-li a-en 1'30) a-en 1'20  a 1 30 u₄
14. (Length together with width) is made span, what is its name?
(u₄ a-en-la lc 2) a₄ su-ka-la 1u 15 u₄ a₄

X • Y = 1'30"+1'20" = 2

16. 2 the surface
a₄

17. So the Akkadian
a₄-sm ac-da-du₄-

16x + y = 30, y = 30

PART C

19. Surface, length and width ACCUMULATED,

1 the surface, 3 lengths, 4 widths ACCUMULATED,
a₄ u₄ u₄ sa₄ u₄ Gar 1 a₄ a₄ 3 u₄ 4 sa₄ u₄ Gar

20. Its 17th to the width appended, 30".
[17] a₄-sa₄ a₄ sa₄ da₄ 30

17y + 3x + 4y = 17'30" = 8'30"

21. You, 30" to 17 go: 8'30" you see
(a₄ je 30 a₄ 1 17 a₄-li-la₄ 8 30 1 17) a₄-nar

17y + 4x = 21y

22. To 17 widths, 4 widths append: 21 you see,
a₄ a₄ s₄ 4 sa₄ da₄ 21 a₄-nar

The coefficient of y is 21,

23. 21 as much as of widths, 3 of three of lengths,
a₄-sa₄ a₄ Gar 3 a₄-li-la₄ 1 a₄

that of x is 3

24. 3 as much as of lengths, 21'30" what is its name?
a₄-sa₄ 3 a₄ Gar 8 30 a₄ a₄ a₄

3x + 21y = 8'30"

25. 3 lengths and 21 widths ACCUMULATED
[3] u₄ 2(1 sa₄) Gar

26. 8'30" you see
8 30 a₄-nar

27. 3 lengths and 21 widths ACCUMULATED
[3] u₄ 2(1 sa₄) Gar
x+1 = X
y+1 = Y
X·Y = (x+y+y) + 1
2

28. 1 to the length append and 1 to the
   width append, MAKE SURROUND:
   (1 a-na) u̇ dag @ 1 a-na sag dag 2 NIGIN-na

29. 1 to the ACCUMULATION of surface, length
   and width append, 2 you see,
   (2 a-) ȧ ṣ a-šu u̇ ṣ a-šu dag 2 ta-gar>

30. (2) the surface. Since length and width,
    those of 2 the surface,
    (2 a-) ȧ ṣ a-šu u̇ ṣ a-šu dag 2 a-ša

31. (1'30' the length together with 1'20' the
    width is made span
   (1,30 u̇ šš-ti 1,20 sag šu-ta-ku-ku

1·1 = 1

32. 1 the appended" of the length and 1 the
    appended of the width
   (1 wu-gū-bi u̇ š 1 wu-gū-bi sag

x·y = 1'30''·1'20''
(identifications)

33. (MAKE SURROUND, (1 you see). 1 and (....)" the
    various (things) ACCUMULATE, 2 you see.
   (NIGIN (1 ta-mar) 1 (....) GL.A UL.GAR 2 ta-mar

34. (3, 21 and 8'30' ACCUMULATE), 32'30' you see.
   (3 (....) 21 (....) 8,30 (....) UL.GAR) 32,30 ta-mar

35. So you ask
   (xi-as)-an ta-ša-al

y = 21 y

36. (....) of the width to 21 ACCUMULATE(F/ION),
   (....).TI sag a-ne 21 UL.GAR-ne

z = 3 x

37. .... to 3, the lengths, raise,
   (....)HIG. A a-ne 3 u̇ a-ša

x·y = 3·21·XY =
1'3''·XY = 1'3''·2
   = 2'6''

38. (1'3'' you see. 1'3'' tī 2, the surface,
    raise:
   (1,3 ta-mar 1,3 a-ne 2 a-ša 1-ša-ne

x·y = 2'6''

39. (2'6'' you see (2'6'' the surface)" 32'30'
    the ACCUMULATION break, 16'15'' you see.
   (2,6 ta-mar (2,6 a-ša) 32,30 UL.GAR ne-pē 16,15
   ta-mar>

(2·y/2)² = (16'15'')² =
4'24'3'05''

40. (16'15'' you see" 16'15'' the counter part
    pose; MAKE SURROUND,
   (16,15 ta-mar) 16,15 gade gar NIGIN
41. 4'24"3'45" you see, 2'6" xxx
4',(24,13),45 ta-ma'r 2,6 (...)  

42. from 4'24"3'45" tear out, 2'10"3'45" you see,  
("s,4,(2),4, 3,4, z1 2,10,3,45 ta-ma'r  

43. What it makes equil. terol? 11'45" it  
makes equil. terol. 11'45" to 16'15"  
append,  
mi-na li-b-si 11,45 li-b-si 11,45 a-na 16,15 daš  

44. 28 you see: from the 2nd tear out, 4'30"  
you see,  
28 ta-ma'r i-na 2-kaim z1 4,30 ta-ma'r  

45. The fig of 3, the lengths, detach, 20"  
you see. 20" to 4'30"   
9 iq1 3-li u8 pu-pur 20 ta-ma'r 20 a-na 4,30)  

46. (20" to 4'30") raise 1'30" you see.  
(20 a-na 4,30) i-it-na 1,30 ta-ma'r  

47. 1'30" the length, that of 2 the surface.  
(What) to 21, the widths, (shall I pose)  
1,30 u8 ša 2 a-ši ni-na) a-na 21 saq (lu-uš-ku-un)  

48. which 28 gives me? 1'20" pose, 1'20" the  
width  
V = 1'20"  
ša 28 i-na-di-na 1,20 saq 1,20 saq  

49. that of 2 the surface. Turn back. 1 from  
1'30" tear out  
ša 2 a-ši tu-ši 1 i-na 1,30 z1)  

50. 30" you see. 1 from 1'20" tear out,  
30 ta-ma'r i-na 1,20 z1)  

51. 20" you see.  
20 ta-(mar)  

* All these restitutions are mine. Restitutions in  
simple [ ] can be regarded as fairly well established,  
those in {()} are reasoned guesses at a formulation,  
the factual contents of which can be relied at.  

**Line 6 quotes the value of the width in a way which  
would usually refer back to the statement, but which  
might of course refer to line 5; in any case, line  
3 presupposes knowledge of the width, and line 5 refers  
to the length as a known quantity.  
† To "AMMAR" translates K1.GUD.GUD, which literally  
seems to mean "to place firmly on the ground/on a  
base". The formulation of line 7 (as supported by  
line 32) suggests that the term may be thought of as  
a logogram for waššum-daš.
The transliteration writes 1. Still, the autography writes a sign after 1 which looks like 20 (and a wedge). That is also the correct result, which is in fact used in line 8.

The exact reconstructions of lines 10-16 are rather tentative, although the mathematical substance is fully well-established thanks to the parallel of lines 28-31. It should be observed that even the extant signs until "1,20 a" in line 14, and the "(b)3" and "(a)3g" of the following lines, are heavily damaged. The remaining traces may but need not correspond to my readings (according to autography and photo). The a3-sum of line 15 is needed, if not necessarily in that place, by the bu-la-bu-bu of line 16, if I am right when reading it as the subjunctive mode of a stativé (cf. lines 30-31).

The transliteration supposes that something is missing in the beginning of the line. The autography indicates that the line is simply written with indentation.

"zu wa-zi-bi in math- SusaText Nr. IX: Ich hatte mich für die Rezension von MDP 34 (von Soden 1964 -- 31) ziemlich gründlich damit beschäftigt und als mögliche Lesung wa-zi-bi als St. const. eines sonst nicht bekannten wurubûn notiert, diese Lesung aber dann als zu wenig gesichert nicht veröffentlicht." (Von Soden, private communication).

"the various (things)" translates BI.A. This presupposes the assumption that the Sumerian suffix BI.A (designating a plurality of different entities) is used as a pseudoSUMERIANUM in a nominal function (as a collective name for the collection of surface, length and width). It is also possible that BI.A stands as a pseudo-grammatical complement to a noun which was lost with the first part of the line.

JHS substitutes "(...) 3-ti aqq" as "ga-la-as-ti aqq" and mistranslates the whole line as "33 <fois> la longueur à 21 fois <la largeur> additionne" in order to get some apparent sense of the restitution. Apart from the mistake of "length" for "width" this mixes up "appending" and "accumulation" -- only the first of these carries a "to" (aqa) between the addends. A possible restitution which accepts the (somewhat dubious) 3-ti in the beginning of the line; which makes mathematical sense; which is as grammatically correct as can be expected in a text loaded with numerograms; and which finally is in reasonable harmony with current usage, would be "17 (....) 7 and 4, of the four (er-bet-ti), widths, to 21, the ACCUMULATION" or "... to 21 ACCUMULATION". In lack of related passages I have, however, preferred to leave the question open.

The transliteration in JHS renders the signs before ga-as as BI.A. The A is in agreement with the autography, but the preceding sign looks very different from the BI of line 33. I have not been able to propose any better reading.

The initial "10" is fully and the final -mar almost fully to be read on the autography, although they are left out in the transliteration. So, a repetition of
Figure 18. The geometrical configurations and operations described in TMS, parts A and B.
The purely explanatory character of part A is revealed already
in line 2, as the surface (which was never given) is referred to
as known ("since ..."), cf. also the restitution of the last
part of line 1. Clearly, we are dealing with one equation
in two (known) unknowns, \( u \times 30 \), \( s \times 20 \), and we are taught the
way to transform it (in fact the same transformation as that of
AN BB67 NOS 1-2, \( xy+xz=xf \), \( y=xz \)). In this way one can make
sense of the "either ... or ... or" of lines 3-5 (0 UL ... 0 UL ... 0 UL),
which governs three alternative ways to explain the transfor-
mation, but which has no place in an interpretation of the text as
progressive argumentation (since the 1'20' created in line 3
is used in line 4, and line 5 repeats the contents of line 4),
and which has therefore puzzled all commentators to the text.

If one follows the text step by step, it turns out that all
Fig. 18 of it can be read as an explanation of the Figure 18A, up to the
end that explains that this is the point out from which problems
containing such equations are to be solved, and finally sums up
the main argument.

Part B deals with the same rectangle, but with a somewhat more
complicated equation, "\( xy+x+y = f \)", and demonstrates how it is to
be simplified "by the Akkadian (method)". It can be followed
on Figure 18B. The method consists in completing the quasi-
gnomon \( xy+x+y \) into a rectangle \( XY \), \( X=1 \), \( Y=y+f \). \( X \) and \( Y \)
are spoken of as "length" and "width" of '2 the surface' (\( TXY \)),
in agreement with the figure.

Denomination of methods is not current in Mesopotamian mathe-
mathematics, and one may wonder what makes the method of part B specifically "Akkadian". Which part of the procedure is it, furthermore, which deserves the label? My guess is that the term characterizes the quadratic completion in general, the basic trick needed to solve mixed second-degree equations. If anything, indeed, distinguishes the Old Babylonian "Akkadian" mathematical tradition from e.g. third millenium Sumerian mathematics, it will be its interest in second-degree algebra. Which more adequate name than the "Akkadian method" could then have been chosen for a trick which, simple as it may look once it is found, was perhaps the starting point for the whole fabulous development of "Akkadian" mathematics; a trick which, when it was first found, will certainly have been noticed as a novelty.

It will be seen from line 14 that the values of both length and width are assumed to be known (though not given in the statement), and that they are used in the didactical exposition.

Part C contains a complete mathematical problem, a set of two equations in two unknown quantities "length" and "width". One of them is precisely the second-degree equation whose transformation was taught in part B, while the other (which can be transcribed \( y = \frac{1}{17}(3x + 4y) = 30 \)) is of the type whose transformation was explained in detail in TMS XVI (above, section VII.3). The values of length and width are still referred to during the solution (line 31), but only for identification, no longer as part of the argument. The identification must refer to something outside the written text, which can hardly be but a material representation more or less similar to Figure 188.

Lines 21 to 26, the transformation of the first-degree equation into \( 3x + 2y = 30 \), must be presumed to follow the pattern from TMS XVI, and hence to be understood as an arithmetical transfor-
mation. Lines 28 to 33 appear to go by "naive geometry". For the next steps, lines 34 to 39, we are unfortunately not in possession of a didactical explanation -- but some argumentation from Figure 18B but similar to the accounting and scaling arithmetic of TMS XVI would at least be adequate, and is perhaps called for in line 27, which appears to connect to the following rather than the preceding section. In any case, lines 39-44 solve the standard problem of a rectangle for which the area and the sum of length and width are known, the "false length" of which is \( x + 3(x + \ell) \)
and the "false width" of which is \( y + 2(y + \ell) \). The method is unfortunately not commented upon -- like the transformation of the linear equation the didactical explanation appears to have been given at an earlier stage, and the understanding now inherent in the vocabulary. Afterwards, the extended "true" length and width (those of "2 the surface") and finally the "true" length and width without extension are calculated (lines 45-51).

The whole tablet reflects a didactical lesson. While part C represents a refined version of a standard problem known from elsewhere (YAT 8520, NNB 1-2 -- cf. note 146), parts A and B are didactical steps toward a particular aspect of the procedure needed to solve the complex standard problem; the other, more general aspects of the procedure are supposed to be known from earlier lessons -- and one of them was in fact explained in TMS XVI, as we have seen.

It has often been assumed that the Babylonian mathematical texts should only be seen as supplementary support for an oral tradition, and that the texts could only be understood by a person who knew beforehand what the whole thing was about. The present investigation shows that this is not absolutely true as
hitherto assumed, if only one knows the concrete meaning of the terminology. But still, the normal texts give the impression that they are a support for a teaching tradition making use of material representations outside the texts themselves, and referring to methods which had to be known beforehand. The material representations have still not been unearthed, and may be irretrievably lost (cf. above, chapter VI). The two Susa tablets, however, show us how the standard methods were taught, and the one just presented appears to refer more clearly perhaps than any other text to the naive-geometric representation.
IX. Summing up the evidence

The investigation has now arrived at a point where a summary of the results can reasonably be made. How far have we come in our understanding of the procedures, techniques and patterns of thought behind the Old Babylonian "algebraic" texts? Chapters IV/ have by necessity been overloaded with details. If all conclusions were to be referred precisely to the single relevant pieces of evidence, the present chapter would make still heavier reading. As the conclusions to be drawn from the material have, however, been presented in scattered form all the way through, I hope that detailed references to the primary material can now be dispensed with.

On the negative side it will be remembered that the traditional arithmetico-algebraic interpretation left so many unexplainable points in the textual discourse that it can be safely dismissed -- cf. most of the texts presented in chapter V. The possibility to make it work by minor corrections and ad hoc assumptions can also be disregarded, because no fundamentally arithmetical interpretation can map the structural distinctions inside the vocabulary. Babylonian "algebra" was not a science about pure numbers and the ways in which they can be put into mutual relation, - be it understood in analogy with Medieval rhetorical algebra as with Thureau-Dangin, Neugebauer and van der Waerden, or through that first-level criticism of the received interpretation which has been expressed by Michael Mahoney.¹⁴⁵¹⁴⁹

Positively, the use of some sort of naive-geometric technique can be regarded as well-established. It fits all details of the textual discourse; it distinguishes operations which have to be distinguished according to the structure of the terminology; it agrees with the apparent metaphorical implications of many terms --
including the puzzling *wásitum*, the "projection". The exact nature of the geometric representation is, however, open to doubt. We do not know to which extent the texts refer to a purely mental representation (though, truly, common pedagogical experience tells that mental geometry presupposes anterior intercourse with manifest geometry) -- and we do not know the means (clay, dust, or even sticks?) which were used to represent geometrical structures, relationships, and transformations manifestly, nor whether such representations should be thought of in analogy with modern geometrical drawings or as mere structural diagrams. These questions were discussed in further detail in chapter VI.

Apart from a two-dimensional extension of the "single false position", the naive-geometrical techniques were only used for problems involving a "surface", i.e. for problems of the second degree. We can list these techniques as follows:

Firstly, there is the partition and rejoining of figures ("cut-and-paste"), which in ordinary "length-width" and "squared line" problems is represented by the bisection and rearrangement of excessive or defective rectangles. In other, genuinely geometrical problems it is used more creatively, and as we shall mention in section X.4 there is evidence for continuity to later interests in the partition of figures.

Secondly, we have the completion technique, the supplementation of a gnomon or a quasi-gnomon into a square or a rectangle. This may be the technique which was spoken of as "the Akkadian [method]" in TMS IX.

Thirdly, we have the "scaling" technique, used e.g. when a non-normalized problem ("$ax^2 + bx + c = y$") is transformed into a normalized problem ("$nx = ax$"), and to be understood perhaps as a change of measuring scale in one direction, perhaps as a proportional change of linear extensions in that direction.

The "accounting" technique may be claimed to have nothing specifically geometric about itself -- and it was indeed set
Forth most clearly in the SUSA text explaining the arithmetical transformations of a linear equation. Nonetheless, the counting of a specific entity (or the measurement of one entity in terms of another entity) is a necessary supplement to the specifically geometric techniques, without which no "analysis" by means of geometry (be it naive or apodictic) can reproduce the results of arithmetico-rhetorical algebra. (The "accounting" and "scaling" techniques are of course closely related.)

Hardly/regular "techniques" but still parts of Old Babylonian naive-geometric methodology are the reasoning by various "false" assumptions and the ability to take any adequate entity of a geometric configuration as that "basic" entity which is to be submitted to the habitual standard operations.

The global picture arising from the use of these techniques and quasi-techniques is the predominance of constructive procedures; only a single pre-established, fixed geometrical standard configuration -- that presented in Figure 2, and visible as a basic grid in Figure 17A -- has suggested itself during the investigation.

The investigation was only peripherally concerned with first-degree techniques. Even on the basis of the restricted material presented here it can be seen, however, that most reasoning about first-degree problems is verbal and basically arithmetical in character. Like second-degree problems, however, problems of the first degree are dealt with by means of "accounting" and various "false" assumptions. Like the second-degree "algebra" the reasoning on questions of the first degree is also concrete, bound probably to mental representations of manifest entities. Hence of course the predilection for "false assumptions", which consist precisely in taking one entity (real or imagined) as a representative for another, normally unknown quantity.

It was recognized already in the early 1930es that Babylonian "algebra" problems were constructed from known solutions. In the
case of the "series texts", where large numbers of problems deal
with the same figure it is also obvious that the user of the
texts would know the solution beforehand. The didactical Susa
texts have now shown us (as it was also apparent from the tabula-
tion in AO 8862 No 1) that even the student would (at least in
certain cases have been told the solution beforehand, which would
permit an identification of the entities involved in the procedure
and also an explanation of the way it works.

The backward construction has traditionally been taken as evi-
dence that the aim of the mathematical texts was the teaching of
procedures and techniques. The insights gained from the improved
understanding of the vocabulary (regarding the use of naive-geome-
tric justifications) and from the didactical Susa texts show us
that the aim was not only technical know-how but also understanding,
"know-why". This helps us grasp how Babylonian mathematics
sole was at all possible at its actual level. If its/social justifica-
tion had been a teaching enterprise dominated by empty rote learn-
ing, from where should it then have got the necessary intellectual
inspiration and surplus?

A summary of the results concerning the details of terminology
would mainly become a repetition of chapter IV, which was in fact
an anticipation of the results established in later chapters. I
shall therefore only refer to Table 1 as the briefest possible
summary of terminological details. On the general level, however,
the somewhat-floating character of the terminology should be
remembered. Only as a first approximation can it be called "tech-
nical". It appears not to have been stripped totally of the conno-
tations of everyday language, nor does it possess that stiffness which
distinguishes a real technical terminology. We should rather
comprehend the discourse of the mathematical texts as a highly standardized description in everyday language of standardized problem situations and procedures, and we should notice that the discourse is never more (but sometimes less) standardized than the situation described. As everyday life contained no second-degree problems (be it the life of a professional scribe, surveyor or accountant), terms taken from everyday language would of course have to be applied differently when describing procedures of second-degree "algebra" than in other texts. In as far as the use in such other texts is taken to represent the "basic meaning", the terms of the "algebra" texts will appear in the quality of standardized metaphors, whence that impression of a technical terminology which is conveyed by standard problems.

The Sumerographic writings inside the otherwise Akkadian mathematical texts presents us with a special interpretative problem. Are they not to be interpreted as technical terminology?

In order to answer this question we have to distinguish different sorts of Sumerographic writing. On one hand we have a restricted number of terms which are invariably written in Sumerian: 𒀀, 𒃾, 𒉗, 𒅸, 𒅷, 𒆠, 𒆠. Even inside this group there is a certain variability, 𒆠, and 𒅷 giving rise to Akkadian loanwords and hence spoken with certainty as Sumerian words, and 𒅸 being often provided with phonetic complements and hence probably spoken in Akkadian (since the complements might be ideographic grammatical complements we cannot be completely sure). None the less, these terms can be regarded as technical and free of everyday connotations -- as it is made especially clear when 𒀀 and 𒃾 used outside the basic representation are suddenly replaced by corresponding Akkadian words (cf. note 75).

Then we have the large number of pseudo-Sumerian writings,
where Sumerograms are used logographically. In/as the logographic meanings of these Sumerograms are not specifically reserved for mathematical texts they are no more/technical than the Akkadian words which they replace (or, alternatively, they are technical with respect to the scribal craft but not with regard to mathematics).

Finally we have a domain of indeterminate extension, that of Sumerograms used as possible alternatives for Akkadian writing but used ideographically. We have met one indubitable instance, viz. Akkadian zî quoted in 7 as an infinitive in TMS XVI, which proves that the category is not empty; but this was an exceptional case, and other instances may be impossible to disclose. Especially the very compact and very ungrammatical Sumerographic writing of the series texts (ungrammatical both from an Akkadian and from a Sumerian point of view) may be suspected to belong here.

The final part of the chapter will deal with two questions of more general character: the relations of our Old Babylonian discipline to the categories of later mathematical thought, and its relation to the intellectual style of its own age.

Throughout this chapter I have spoken of Old Babylonian "algebra", not algebra. But was Babylonian "algebra" an algebra? Put in this form the question will of course have to be answered by in itself a definition, which is not a very fruitful way. We shall learn more by asking, in which respects Babylonian "algebra" was similar to Medieval or post-Renaissance algebra?

We should start from the outside, observing the uses to which the Babylonian discipline was put -- and not put. In later times, algebraic techniques have been used to find the solution to proper problems which could not be solved by direct/. We have no Babylonian texts which suggest such uses of the naive-geometric "algebra"; on the contrary, the specious problems which had to be constructed in order to give occasion for the display of "algebraic" second-
degree techniques suggest that no real uses were known (the abundance of realistic manpower- and brick-problems demonstrate that the eventual Babylonian school-masters did nothing to hide on/real-life importance of their teaching). "Algebra" never served to find a numerical value unknown in advance. In that respect its function was very different from that of algebra.

Recognition of this difference should not force us into the opposite extreme, and should not make us believe that naive-geometric "algebra" was nothing but an investigation of squares and rectangles, a peculiar sort of geometry. In chapter I I introduced the concept of a "basic conceptualization". The u e and s e are indeed basic in the sense that they are used to represent other quantities, the arithmetical relations between which can be mapped by the relations between the lengths and widths of rectangles. In YBC 6967 we have seen how a pair of numbers with known product and difference was represented by the dimensions of a rectangle, made visible in the text by the explicit reference to a "surface". Other texts would show a wide variety of quantities being represented as linear quantities, more or less explicitly mentioned. Especially interesting are certain cases where the text appears to distinguish between the linear extensions of a real figure, perhaps supposed/to be situated in the terrain, and the corresponding extensions of a representing figure (drawn perhaps in the dusty schoolyard), even though the two coincide numerically (159). Naive-geometric analysis of quadrangles is hence used as a means to solve problems from other domains -- be they artificial and the solutions known beforehand to exist as regular numbers. Though "algebra" was in all probability not used instrumentally in non-artificial situations, it was obviously taught as a virtual instrument (156). Endscope

In virtual use, "algebra" was hence related to real algebra. Can a similar claim be made for its "essence", its internal structure and characteristics? In a criticism of the unreflected
use of the modern term to characterize a Babylonian discipline. Mahoney has listed three characteristic features of developed algebra (157). Firstly, the employment of "symbolism for the purpose of abstracting the structure of a mathematical problem from its non-essential content"; secondly, the search for "the relationships (usually combinatorial operations) that characterize or define that abstractness structure or link it to other structures"; thirdly, "and absence of all "ontological commitments".

Taken at the letter, and allowing only for divergence "by degree rather than kind", these features are only valid for post-Vietan algebra understood as a scientific discipline. Already Medieval or more recent practitioners' algebraic calculation will only deserve the label "algebraic approach". In the same Old Babylonian strict language, "algebra" is algebraic "in approach": It cannot be claimed to possess a real symbolism; still, even if the uē and sēg are no more symbols than the Diophantine ἀριθμὸς or the Medieval thing, their use as ingredients of a "basic representation" serves precisely if only implicitly "the purpose of abstracting the structure of a mathematical problem from its non-essential content". Secondly, a number of systematic texts (especially among the Šešû texts, but even BN 13901 can be mentioned) are in fact systematic investigations of the relationships characterizing the uē-ēg-structure. Only the third criterion is not fulfilled even tendentially -- unless we will claim that the use of a common basic representation is already virtual abstractness.

The "essence" of algebra can also be approached in another way, which links the beginnings of scientific algebra more clearly to the Medieval Art of Algebra (and to the practitioners' algebra of the Modern era). In his Introduction to the Analytic Art, in
which Vieta aimed at bringing to light the hidden gold of algebra and al-muqabala, he found the true essence of that art in the Ancient method of analysis, "die Annahme des Gesuchten als bekannt und der Weg von dort durch Folgerungen zu etwas als wahr Bekanntem". This is exactly what we teach school children to do when solving an equation: "You treat x precisely as if it were an ordinary number". Apart from the known values used for identification purposes during explanations (but not as steps in the mathematical argument, cf. THS IX, part C) it is also a precise description of the Old Babylonian procedures. In this respect, too, Old Babylonian "algebra" is therefore algebraic, or at least characterizable as "naive-geometric analysis".

Was "algebra" then an algebra? If we apply Mahoney's criteria, it was not. Babylonian mathematics differed more than in degree from the discipline founded by Vieta and continuing through Descartes and Noether. But it was "algebraic", belonging in full right to any family which is able to encompass both al-Khwārizmī, Cardano and Noether. Anybody using confidently the expression "Medieval algebra" can with equal confidence speak of "Babylonian algebra".

Instead of relating our subject to categories of later times we may compare it to the general cognitive style of its own time, thereby regarding it as one aspect of the thought of its times, on an equal footing with others.

In their introduction to a famous "essay on speculative thought in the Ancient Near East" H. and H.A. Frankfort characterize it as "mytho poetic". There are several facets to the concept, but its main implication is that the phenomenal world is no object, no "it": it is a "thou", an animated individual. In as far as this
is an adequate description it excludes a scientific cosmology in the modern sense, a cosmology extrapolated under theoretical guidance from rational experimentation and hence in the final instance from technological practice (I agree with any critical mind who finds this description short-circuited). In this sense, it is true, we find no scientific cosmology in Ancient Mesopotamia; in the same sense it is indeed difficult to connect a scientific cosmology to any poetical or religious world-view, and so far it is therefore not obvious that the domination of cosmology by myth should imply that Ancient Mesopotamian thought in general be mythopoeic.

Now, not everything in Babylonian thought was speculative; much of it was founded on social practice or on technological practice. In both of these, and especially in the latter, the object-aspect of the external world (which under this view is not just "phenomenal") must be expected to impose itself. It is therefore not astonishing that it seems "difficult to accept [mythopoeicy] as an adequate characterization" of "the intellectual adventure of ancient man" as "documented in the corpus of administrative, commercial, technical and other genres."

Our algebraic texts constitute another exception to the presumed mythopoeic rule. Truly, AO 8862 carries an invocation of the scribal goddess Nisaba on its edge; but this and other similar inscriptions is totally isolated from the rest of the text, which a "thou" treats its subject not as having the "unprecedented, unparalleled, and unpredictable character of an individual, a presence known only in so far as it reveals itself," but as a fully predictable and comprehensible object. No wonder, since Babylonian algebra was definitely not "speculative", i.e. "regarding", but active, technical construction. According to H. and H.A. Frankfort's dichotomy it is "modern", dealing with lengths, widths and sur-
faces and with its problem-situations as "objects and events [...] ruled by universal laws which make their behavior under given circumstances predictable", and which "can always be scientifically related to other objects and appear as part of a group or a series"[66].

This does not mean that Babylonian mathematics (and technical thought in general) was modern, only that its difference from modernity cannot be grasped by the Frankfort dichotomy. Nor should the secular rationality of Hammurapi's "Code" make us mistake this collection of concrete decisions for an abstract, general law-book in the style of Roman law[66]. A recent investigation of the cognitive character of Babylonian divination science[67] tries to get beyond such mistakes through reference to Lévi-Strauss's distinction between "hot" and "cold" societies, between the "savage" and the "domesticated" mind, between "the science of the concrete" and that of illustrated by the distinction "abstract thought", between the "bricolier" (a cross-breed between the "tinkerer" and the "Jack of all trades") and the engineer[68].

In the Lévi-Strauss illustration, engineering technology is thought of as developing specialized tools for the job to be done; the bricolier, on the contrary, takes what happens to be at hand and fits it together as best can be done. "Domesticated" science and thought is seen analogously as building on abstract concepts; the "savage mind", on the other hand, classifies the categories and oppositions of e.g. their social world using pre-existent entities as classifiers and analogies[69]. While concepts are "wholly transparent with respect to reality", meaning nothing but their conceptual content, a pre-existent concrete entity used as a symbolizer is a sign, preserving to some extent the cultural meaning it possesses in itself and importing it to those other entities for which it is used as a classifier[70] (being a member of the "arrow clan" may imply swiftness!).
In his investigation of the Babylonian/omen literature, Trolle Larsen comes to the conclusion that many features (the search for classificatory order and the postulate of direct causation, partly built on recorded experience and partly on analogic thought) can be described as "savage". Other features of the omen literature are (from its Old Babylonian beginnings) better described as "semi-domesticated": The intent to engineer the future, the attempt to make exhaustive listings of all possible omen (which presuppose writing, a main domesticator) and the way in which lacunae in the empirical record are filled out by means of abstract, logical rules -- which are in fact formulated in a Neo Assyrian compendium. All in all, however, the global logic of the field was such that the apparent steps toward "domesticated science" could lead nowhere.

How are we then to regard Old Babylonian mathematics? Is it also "lukewarm", blocked midway between a neolithic "cold" society and our modern "hot" world?

Several features, at least, look "savage". It was claimed time and again in the preceding chapters that a pattern of thought was "concrete", which sounds very much like the classification by means of pre-existent, concrete entities used as signs. But let us look at the concrete argument in VAT 8399 No 1. In this case "concreteness" means that the mathematical structure is thought in terms of the real entities involved -- there is no distinct, concrete signifier, no sign imparting to the meadows its own meaning. "Concreteness" means simply "absence of any explicit abstract signifier or abstract calculating scheme" (no x or ἀπλά μήκος, no standardized "double false position").

In second-degree problems like those of BM 13901 or AO 8862 (the "basic representation" itself), we see the same sort of concreteness. "Naive geometry" consists precisely in taking geo-
metrical entities at their phenomenal face value, without sub- 
mitting them to theoretical reflection through which their mutual 
relationships might be formulated as abstract principles.

In cases where something else is dealt with by means of a 
mapping on the basic representation, be it the number pairs of 
a table of reciprocals, prices, or real linear extensions, we 
seem to come closer to the use of concrete entities as signs. 
Even here, however, we should take care. There is no hint that a 
price represented through a length has anything in common with 
that line -- except, precisely, the relevant characteristic, the 
measuring number. No text whatever suggests anything similar to 
the swiftness of the arrow clan. On the contrary, the representa-
tion is normally only visible through the designations of the 
operations performed ("breaking", "making span", etc.). Only 
occasionally do we find a "surface" or a "true length", etc. 
In its function, the basic representation can be regarded as 
an abstract instrument.

Places where the description of "savage thought" is 
really relevant for Old Babylonian algebra are its terminology, and 
hence its operations. Like Levi-Strauss's "concepts", technical 
terms are "wholly transparent", meaning nothing but their direct 
technical implication. They have no connotations. Like his "signs", 
descriptive metaphors (even when used in a standardized way as 
everyday long as the situation itself is standard) carry a load of conno-
tations, causing e.g. its users to "tear out" rather than "break off" 
a square from another square. The terminology being only partly 
technicalized, we might characterize it as "semi-savage".

A second "semi-savage" aspect of Old Babylonian algebraic meth-
ematics is constituted by the series texts. As I have not dealt 
with them above, I shall only state briefly that the listings 
of large numbers of variations on the same type of equation is
a parallel to the way all possible liver shapes are listed in the omen lists, and to the lexical lists. But it is no perfect parallel; while the lists are first of all additive and aggregative listings, introducing hierarchical ordering only in so far as this reflects "the surrounding highly stratified society," the series texts are constructed in main sections, first order subdivisions, and cartesian products of second-order subdivisions.

In the case of the omen text, the Neo Assyrian compendium formulating explicit, abstract rules was an unprecedented innovation (at least as far as the written record has been excavated). In mathematics, the corresponding step can be demonstrated to have been taken already by the late old Babylonian period, viz. on the Susa text TNS XVI, which furthermore looks very much as a written documentation of a sort of didactical explanations which would normally be given orally. It does not in itself constitute theoretical reflection on abstract principles, and it is thus no step leading automatically to abstract, deductive mathematics. From which it is a starting point/s critically inquisitive intellectual environment might have been able to proceed indefinitely long. Sticking to the cold-hot metaphor we may say that Old Babylonian algebra was after all not only "lukewarm" but also inflammable. Of the discipline further development/was not blocked by an immanent intellectual structure reflecting the overall social and intellectual climate, as was the case of divination science; the blocking factors resided directly in global social and intellectual conditions: The scribal school was only moderately inquisitive and definitely not critical; the prime reason for interest in mathematical professional pride and knowledge beyond the requirements of direct utility was/social prestige rather than curiosity and openness to the infinite possi-
ibilities of an unknown world. Furthermore: By the end of the Old Babylonian era, the scribal environment changed socially and intellectually, cutting off even the supplies for that sort of mathematical research which had been undertaken until then.
X. The legacy

So, after the end of the Old Babylonian era, second-degree algebra vanishes from the documentary horizon for many centuries -- as do in fact all traces of mathematics teaching. That does not mean, however, that Old Babylonian mathematics was a complete mathematical dead-end without consequences for later mathematical cultures. On the contrary: though rarefied for a millennium below the level of archaeological visibility, the Old Babylonian tradition was to exert its influence on several of the sources of Modern mathematics.

Before looking directly at the evidence for such influence we shall, however, investigate yet another Old Babylonian text, one in which the conceptual dynamics of Old Babylonian algebra can be glimpsed.

X.1. A possible shift in the conceptualization: IM 32301 No. 2
(Barbier 1950a, improved transliteration in Grundach & von Soden 1963:252f)

The text in question is from IM 32301, perhaps the youngest of the (northern) Tell Yamgal mathematical tablets. It deals with a real geometric trapezium, and reduces the problem to one of "surface and squared lines equal to number". Besides being a beautiful specimen of "representation", the text is interesting because of its deviations from normal usage, which suggest a tendency toward changing or looser conceptualizations. It runs as follows (the marginal drawing is not in the tablet):
16. If to two-thirds of the accumulation of the upper width
\( \text{sumer a-na ši-ni-ip ku-su-ta saa wu-li-tim} \)

\( V_1 - (u-x) \times 10 \times x(z20) \)

17. and the lower, 10, to my hand, I have appended; 20 the length I have built. The width
\( \text{ù ša-aq-li-tim 10 a-na qa-ti-la daš-nu 20 wu ša-mi saa} \)

\( w-x \div 5 \)

18. (\(...\)\) the upper, over the lower 5 goes beyond.
\( (e-li) a-li-tum e-li ša-aq-li-tim 5 l-še-at \)

Putting \( w+x = z \times 2'30' \)

\( x = \frac{\frac{z}{2} + 10}{2} \)

19. The surface is 2'30'. What are your lengths?
\( \text{You, by your saying, 5 which it goes beyond} \)
\( a-ša 2'30 mi-nu-um a-ša ša-e tuk-sú-da 5 ša e-te-tu \)

\( (l/2) \times (\frac{z}{2} + 10) = z'30' \)

20. 10 which you have appended; 40' of the two-thirds,
my factors of both (\(...\)\) inscribe:
\( 10 ša tu-is-bu 40 ši-ni-pé-tim a-ra-na ni-a-ti-a lu-pu-ut-ma \)

or, with an adequate choice for a:

\( (l/2) \times (2+2a) = \)
\( (\frac{z}{2})'2'30' = 3'45' \)

21. The 1 of 40' of the two-thirds detach; 1'30'
you see. 1'30' (\(...\)\)
\( i-gi 40 ši-ni-pé-tim pu-tu-ur-ma 1,30 ta-mar 1,30 \)
\( še-pé(\?)-ma \)

22. (\(...\)\) to 2'30', the surface, raise 3'45'
you see.
\( 4'5 ša-mar 45) a-na 2,30 a-ša ša-li-ma 3,45 ta-mar \)

23. 3'45' double; 7'30' you see. 7'30' your head
\( 3,45 e-gi-ma 7,30 ta-mar 7,30 ri-ga-ka \)

24. may retain. Turn back. The 1 of 40' of
the two-thirds detach
\( ši-li-li tu-ur-ma i-qi 40 ši-ni-pé-tim pu-tú-ur \)

Reverse

1. 1'30' you see. 1'30' break; 45' you see; to
10 which you have appended
\( 1,30 ta-mar 1,30 še-pé-ma 45 ta-mar a-na 10 ša tu-is-bu \)

2-4. raise. 7'30' you see (\(...\)\)

\( i-li-ma 7,30 ta-mar 7,30 ri-ga-ka ši-li-li \)

\( tu-ur-ma i-qi 40 pu-tú-ur-ma 1,30 ta-mar 1,40 še-pé-ma \)
\( 45 ta-mar a-na 10 ša tu-is-bu ši-li-ma 7,30 ta-mar \)
5. 7°30' the counter(...) port lay down; Make span:
7°30 me-eh-ša-ki-na i-di-na št-ta-li-il-na

(2) 7°30' you see. 56°15' to 7°30' which your hear
56,15 ta-mar 56,15 a-na 7,30 ša ši-ši-ka

7. retains append: 8°26'15' you see. The equilateral
ű-ka-ši-ši-il-ši-ša 8,26,15 ta-mar ba-se-e

8. of 8°26'15' make come up: 22'30'
its equilateral; from 22°30'
8,26,15 šu-li-ša 22,30 ba-sšu-šu i-na 22,30

9. the equilateral 7°30', your toklitum, cut off,
ba-se-e 7,30 ta-ši-ši-ta-ta šu-ra-ša

10. 15 the left-over. 15 break: 7°30' you see.
7°30' the counterpart lay down:
15 ša-ta-ta-ta 15 še-pé-na 7,30 ta-mar 7,30 me-šš-ša-ša
i-di-na

11. 5 which width over width goes beyond break:
5 ša šag e-li šag i-ta-ši še-pé-na

12. 2°30' you see. 2°30' to the first 7°30' append
2,30 ta-mar 2,30 a-ne 7,30 ša-ši-ši-ši-il-ši-il-ša

13. 10 you see; from the second 7°30' cut off.
10 ta-mar i-na 7,30 ša-li-na šu-ra-ša

14. 10 the upper width; 5 the lower width.
10 šag e-li-ta-ta 5 šag ša-ši-li-ta

Proof:
15. Turn back: 10 and 5 accumulate, 15 you see.
ta-uš-me 10 ša ku-mu-ši 15 ta-mar

16. The two-third of 15 take: 10 you see, and 10
append:
ša-ni-ip-at 15 ša-gé-na 10 ta-mar ša 10 ši-li-il-ša

17. 20 your upper length. 15 break: 7°30'
you see.
20 Ša-ša e-li-ša 15 še-pé-na 7,30 ta-mar
18. 7'30" to 20 raise: 2'30", the surface, you see.

   7,30 a-na 20 i-ši-ma 2,30 a-ša ta-mar

19. So the having-been-made.

   ki-a-en na-pu-šum

8. I.e. a number 10 which is "at my disposition" without being defined in relation to the figure.

9. The text contains a number of repetitions, other erroneous insertions etc. due to faulty copying. Those of obv. 18 and rev. 1 were already pointed out by Baqir (Plate of obvs. 21-22 and rev. 2-4, the first of which has been induced by the phrase "1,30 ta-mar 1,30" common to obv. 21 and rev. 1, while the second is provoked by the "7,30 ta-mar" common to obv. 23 and rev. 2) follow from analysis of the procedure.

   The reading of zo as a homophonic mistake for zü in obv. 19 was given in von Soden 1952a:49. That of TUK as dug was suggested by Baqir (1950a:146).

10. "Factors of both" is a tentative translation of hamanum, a plural form known from nowhere else. The term is an epithet to 46', which multiplies the sum of the widths. The term thus appears to suggest two (identical) factors multiplied the numbers of a sum. In agreement with this, von Soden (1952a:50) suggests conjecturally the word to be a loanword from Sumerian ar-ta-man, "times"-"two", i.e. "factors of both".

11. The "equilateral" of rev. 7-9 is written in syllabic writing. In rev. 7 and 9, the form is BA.S.E, indicating that the form normally written ba-a-si₄ (which alternates with i₂-bi₄) was pronounced in Sumerian. (In a similar fashion, the text writes a syllabic i₂-gi₁ instead of the normal i₂-gi₁). In rev. 8, the form is a nominative with suffix, ba-mu-šu, suggesting an Akkadianized form beside. The accusative form in rev. 7 could in principle be a construct state of the same form, but the genitive in rev. 9 cannot, since the rest of the text is written with full imitation -- it must render a genuine Sumerian pronunciation of the term.

   Both forms confirm (as does the homophonetic shift from a₁₁ to a₁ in certain texts) that the term was not read as a logogram for an Akkadian word (aššarrum being the normal assumption), at least not when used for the extraction of a square-root.

   In AO 17264 (late Old Babylonian or early Kassite) the forms ba-si₂-šu and ba-ši₂-šu are found (KMT I, 127). Even here, the equilateral is "asked for" (ailum).
Before drawing any conclusions from the way the text formulates its subject-matter we should of course make sure that this subject-matter is understood correctly. Is the interpretation in the marginal commentary adequate (apart from the anachronism inherent in the use of modern algebraic symbolism)? Should we not instead expect that the problem was seen as one in two unknowns (a "length-width"-problem) the product and difference of which are known (x and x=2a, in the symbolism of the margin)? Or, if it is to be understood in terms of one unknown ("surface and squared lines"), is the average width \(\frac{u+x}{2} = z/2\) not the entity which would normally be chosen by a Babylonian?

Both answers should probably be answered by "yes"; we should perhaps expect the problem to be comprehended in two unknowns, and if not, the average rather than the aggregated width would be a normal Babylonian unknown. But in the first case we would also expect that the difference between the two be really calculated; instead, the scaling factor 1'30" is bisected before the multiplication is performed, without any other reason calling for that sequence of operations. In the second case, the operation in obv.23 would have been a "raising", the normal scaling multiplication (cf. section V.5, BM 13901 No 3), and that of rev. 10 would have been a reverse scaling. Instead, the first is a "doubling" and the second a "breaking", concrete operations which indicate that operations belonging with the standard procedure are only found from obv. 24 to rev. 9, and thus that the sum of the widths, i.e. the 15 found in rev. 9, is the quantity looked for in that procedure. All normal Babylonian habits notwithstanding, the marginal commentary appears to map the original procedure.

If we look at the formulation of the text, it is obviously close to the style known from Old Babylonian algebra in general,
so much so, in fact, that lack of feeling for the stylistic implications of the naive-geometric procedures (most notably the identification of the 7,30 of rev. 9 as a *taklimu*, i.e. as the same as that of rev. 5) has prevented earlier investigators of the text from identifying correctly the ditto-graphics of obv. 21f and rev. 2-4.

Apart from the erroneous repetitions (which are obviously due to copying errors and which therefore presuppose the existence of a more correct original) and the syllabic writings of Sumerian terms there are, however, certain deviations from normal usage which can hardly be explained unless we assume some slackening of normal conceptual habits.

Firstly, the term "building" is employed in obv. 17 when the length is explained to be equal to the sum of the widths and an extra amount of 10. It is not excluded that a constructive procedure is still intended -- but in that case a mental construction is more plausible than an actual drawing. In any case, the formulation deviates from a normal usage which appears to be strongly bound up with specific procedures.

Secondly, a "counterpart" turns up in rev. 10 in a most unusual function. Normally, it is seen in length-width-problems when two sides forming an angle of a completed square are "laid down", for addition and subtraction of the *taklimu*, respectively (cf. YBC 6967, -section V.1)

In the present case, addition and subtraction of a semi-difference is still meant, - but if a geometrical configuration is at all thought of, it is different, the "original" and the "counterpart" being opposing widths of a rectangle, which the addition and subtraction are to transform into a trapezium.

These peculiarities do not prevent a naive-geometric interpretation. Moreover, the "doubling" in obv. 23 suggests the use of a
Figure 19. The geometrical interpretation of IN 52301 No 2 suggested by the parallels in VAT 7532 and VAT 7533.
procedure related to a trick used in the two tablets VAT 7532 and VAT 7535 (both in HKT). The suggested procedure is shown in Figure 19: The step of obv. 21-22 corresponds to a scaling in horizontal direction (the first transformation, A→B). The doubling in obv. 23 is a real repetition, transforming the trapezium into a real rectangle (B→C), viz. a "surface (of a square) with 15 squared lines". The sequence of operations is, however, remarkable. If the geometrical procedure had been performed physically, it would have been natural to make the doubling first, and the scaling afterwards. The actual sequence appears to indicate that a more purely arithmetical understanding of the underlying structure, where the sum of the widths is aimed at as an unknown (in the first transformation) before it is actually produced (in the second transformation).

The deviant use of the term "building" was already mentioned as an indication pointing in the same direction. The implications of the peculiar use of "counterpart" in rev. 10 are more indefinite, and the most that can be said is that an otherwise strict conceptual structure appears to be loosening (especially if we notice that the term is also used in a more/orthodox way in rev. 5). The way the text regards the "equilateral" is, however, yet another indication that an arithmetical conceptualization is present: It is definitely no entity producing a square -- it is something which "comes up", i.e. a numerical result.

The awareness of a homomorphism between geometrical and arithmetical procedures need not have been greater with the author of the present text than with the authors of more orthodox, somewhat older texts. The latter, however, formulate themselves strictly within the geometrical conceptualization. This strictness of language has either been regarded as superfluous or has not been
understood by the present author. In both cases it is justified
to speak of a loosening of the conceptualizations and of an
opening toward explicit arithmetical understandings.

X.2. Seleucid arithmetization: BM 34568 No. 9 (MKT III, 15)

Further developments of this opening toward arithmetic are seen
in the algebra problems of the Seleucid era. A simple instance
is found in BM 34568 No. 9, the very problem which was used in
Chapter I to demonstrate the ambiguities of current translations.
In transliteration and conformal translation, the text runs
like this:

Obverse II

\[
\begin{align*}
1. \ \text{Length and width accumulated are} & \quad 14, \text{and } 48 \text{ the surface.} \\
& \text{u₃₄₅₆₇₈₉} \\
& \text{I know not. 14 steps of } 14, \text{ } \\
& 3'16". 48 steps of } 4, \text{ } 3'12". \\
& \text{mu nu-xu₂₃} \\
2. \ \text{From } 3'12" \text{ (to) } 3'16" \text{ go up: } 4 \text{ remains.} \\
& \text{What steps of } \text{what?} \\
& \text{3,12-ta } 3(1)6 \text{ nina₃} \\
& \text{ri-ni₃} \\
& \text{mu nu-ga GAM} \\
& \text{mu-ni-1} \\
& \text{3,12-ta } 3(1)6 \text{ nina₃} \\
& \text{ri-ni₃} \\
& \text{mu nu-ga GAM} \\
& \text{mu-ni-1} \\
3. \ \text{shall I go so that } 47 \text{ 2 steps of } 2, 4. \\
& \text{From 2 (to) 14 go up: 12 remains.} \\
& \text{lu-re₃₄₅₆} \\
& \text{3} \\
& \text{4 GAM} \\
& \text{2} \\
& \text{2-ta} \\
& \text{14} \\
& \text{nina₃} \\
& \text{ri-ni₃} \\
& \text{12} \\
& \text{TIMES } 30', 6 \text{ the width. To odd } 6: \\
& 8, 8 \text{ the length.} \\
& \text{12 GAM} \\
& \text{30} \\
& \text{6} \\
& \text{sag} \\
& \text{2} \\
& \text{e} \\
& \text{6} \\
& \text{ta-tip-pi} \\
& \text{mu} \\
& \text{u₃₄₅₆₇₈₉} \\
4. \ \text{shall I go so that } 47 \text{ 2 steps of } 2, 4. \\
& \text{From 2 (to) 14 go up: 12 remains.} \\
& \text{lu-re₃₄₅₆} \\
& \text{3} \\
& \text{4 GAM} \\
& \text{2} \\
& \text{2-ta} \\
& \text{14} \\
& \text{nina₃} \\
& \text{ri-ni₃} \\
& \text{12} \\
& \text{TIMES } 30', 6 \text{ the width. To odd } 6: \\
& 8, 8 \text{ the length.} \\
& \text{12 GAM} \\
& \text{30} \\
& \text{6} \\
& \text{sag} \\
& \text{2} \\
& \text{e} \\
& \text{6} \\
& \text{ta-tip-pi} \\
& \text{mu} \\
& \text{u₃₄₅₆₇₈₉} \\
5. \ \text{12 TIMES } 30', 6 \text{ the width. To odd } 6:
& 8, 8 \text{ the length.} \\
& \text{12 GAM} \\
& \text{30} \\
& \text{6} \\
& \text{sag} \\
& \text{2} \\
& \text{e} \\
& \text{6} \\
& \text{ta-tip-pi} \\
& \text{mu} \\
& \text{u₃₄₅₆₇₈₉} \\

\]*

"accumulated" translates GAR, which is certainly an abbreviation for GAR-gar, not as in Old Babylonian texts a logogram for SAGURU, "to pass".

NAME translates MU, used logographically for SUMUM.
Thureau-Dangin's interpretation as a logogram for ASSUR, "since" (THB, 59) is possible, but it does not fit the context. Neugebauer's interpretation "name" is, on the other hand, confirmed by the Susa text IMS IX.
First of all we observe that certain parts of the vocabulary are continuous with that of our Old Babylonian texts: "length", "width", "surface", "name", "steps of". All except "steps of" belong on the level of algebraic problems, not on that of mere computation. We can therefore be sure that we are really confronted with a descendant of the Old Babylonian algebraic tradition, in spite of the silence of all sources between c. 1600 B.C. and c. 700 B.C.

The next observation will be that of thorough change on all levels, in spite of the continuity. It goes down to the choice of Sumerograms: n i m, which in Old Babylonian texts designates
Figure 20. Two all-purpose figures which may support all the second-degree problem solutions of BM 34568. The upper figure will be recognized as a familiar justification of the Pythagorean theorem. For use of the lower figure one shall remember that the central square equals the sum of the upper left and the lower right square \((c^2 = a^2 + b^2)\). In problem 12, the equality of the lower right square and the central gnomon will have to be used explicitly.

The upper figure is seen to contain Figure 17A, the one constructed for AO 8862 no. 3. It will be remembered (see above, note 138) that the same configuration appears to be used in two other Old Babylonian problems.
a multiplication of the "raising" class, presumably for the word "ullûm" (cf. note 39), is used now for the stepwise counting of a difference, presumably as a logogram for "slûm". In part, at least, the Sumerianization of mathematical language appears not to have been continuous over the silent millennium.(78)

The discontinuous Sumerianization carries implications for the nature of the transmission, which appears to have taken place in a practitioners' environment rather than a scholarly institution. As far as the conceptual structure of Seleucid algebra concerns it is less important. Here, the absence of all traces of constructive thought and not least the purely arithmetical formulations are fundamental. Subtraction has become a pure counting process, instead of a concrete process described metaphorically in physical terms ("tearing out", "cutting off", etc.). Only one multiplication operation is left, described by the term of multiplication tables (i.e. as a repeated counting), when not by the ideogram GAM, the separation sign used apparently as a purely visual symbol. Bisec-
tion is no special operation, but only a multiplication by 30° -- and the square-root is explicitly asked for as the solution to the problem $x \cdot x = n$. Two additive processes appear to be present, but the one corresponding to "appending" can no longer be identity-conserving, since it is often (though not here) symmetrical with respect to the addends. No doubt, therefore, that the conceptualization of the problem is completely arithmetical.

As discussed at some length in chapter I, an arithmetical conceptualization does not exclude a geometrical method and justifica-
tion. This combination is precisely what is found in al-Khwarizmi's justifications. A figure which would serve to solve the problem was shown in Figure 2, and the same figure and a generalized version will in fact explain all problems of the tablet (except one dealing with mixture of metals and one dealing with a rectangle of known

Fig. 20 proportions),- see Figure 20. Moreover, even the more specious
procedures are easily argued from the two all-purpose figures --
and in one case, that of No 13, Neugebauer feels obliged to have
recourse to Figure 20 B (79) in order to explain why the pro-
cedure is at all meaningful. On the other hand, several of the
solutions are very difficult to follow unless one uses either
gometric support or written, symbolic algebra -- purely rhetorical
methods will not do. It is therefore reasonable to assume that
the method of Seleucid second-degree mathematics remained geome-
tric, in spite of the arithmetization of its conceptualization
(though probably "synthetic" rather than analytically constructive).

It is tempting to see the arithmetical conceptualization as the
final outcome of a natural process already begun during the late
Old Babylonian period. Secular use of the same procedures would
grind off everything superfluous and leave back only the essential
structure, which is indeed arithmetical. Before accepting this
sole and sufficient explanation we should, however,
be aware that one other factor was also at work -- and perhaps
even a third circumstance should be taken into account.

The indubitable extra factor is the specific scholarly environ-
ment of Seleucid mathematics: the great astronomical centre of
Uruk (180). The enormous numerical calculations performed in this
centre may well have made the local scribes more inclined toward
arithmetical thought than less specialized practitioners of the
algebraic art (whoever they may have been -- but as we shall see
below, such practitioners must have existed).

The possible extra factor is cultural cross-fertilization.
Seleucid Uruk was part of the Hellenistic melting-pot, and links
back to Old Babylonian traditions should therefore not be taken
to exclude combination with other links. In another branch of
Seleucid mathematics, viz. mensurational geometry, a definite
break with Old Babylonian methods and a striking parallel to
Alexandrian geometry is clearly visible (181).

In the procedure of our problem there may also be a suggestion of cultural import. All corresponding Old Babylonian problems find the semi-sum and the semi-difference between length and width, even those which appear to make use of the same geometrical configuration. In the present case, the total sum and difference are found. There is no inherent reason for that change. In a group of more orthodox second-degree problems in the Seleucid tablet AO 6484, dealing with igu₃-igib₃-pairs with known sum (182) (as far as mathematical structure concerns no different from the present problem), we find indeed the traditional semi-sums and semi-differences, together with a terminology which is about as arithmetical as that of the present problem (183).

A purely autochthonous development would probably have affected the method of all isomorphic problems similarly. It is therefore plausible that the specific methods of BM 34568 were introduced together with a specific cluster of length-width-diagonal-problems during the dialogue of scientific cultures.

It is not possible to identify the eventual interlocutor. Similar interest are found in China, in the Nine Chapters on Arithmetic (184). But they are also found in the Graeco-Roman world (185), and in neither case are the similarities complete or fully convincing. Furthermore, the Hellenistic era was one of wide-range cultural connections, from China to Magna Graecia. The suggestive similarities can at most be taken as indications that mutual inspiration took place, and that Babylonia was probably not the only focal point for "algebraic" investigations of geometric figures.
X.3. Babylonian influence in Greek mathematics?

The eventual foreign inspiration of Seleucid algebra is difficult to trace precisely. So are also the eventual inspirations flowing the other way during Antiquity and the early Middle Ages. Certain suggestions can be found, however, in Greek sources pointing to inspiration though hardly to direct descendency.

The idea of inspiration from Babylonian algebra to Greek "geometric algebra", i.e. the geometry of *Elements* II etc., is as old as the discovery of Babylonian second-degree algebra. Since the late 1960s it has been submitted to severe criticism (186), mainly because the Greek geometry of areas is a coherent structure of its own which is not adequately explained as a "translation" of an arithmetico-rhetorical algebra, of which it is neither an isomorphic nor a homomorphic mapping.

A naive-geometric reinterpretation of Babylonian algebra changes much of the foundation of the debate (187). If we recognize further that the structure of Greek geometry is the result of a process and not identical with the structure of its possible inspirations, the question of Babylonian inspiration of Greek mathematics is completely open again.

This is not the place for a thorough investigation of the problem, which I approach elsewhere (188). I shall just point to an observation which put me on the track. The much-discussed term ἀνάγωγος has given rise to precisely the same ambiguities as the Babylonian *mīthartum*. In some contexts it seems to mean "square-root" or "side of square", in others it is the square itself.

As in the Babylonian case, the apparent ambiguities are eliminated if we read the term as "a square identified by (and hence with) its side". The normal Greek habit is to identify a figure with its area; as with us, a square designated τετράγωνον has a side and
in its area. The δύναμις is thus a foreign flower in the Greek conceptual garden.

Investigation of a variety of (mostly early) sources suggests that the term was not only used in theoretical geometry but also by calculators -- seemingly in connection with some sort of algebraic activity (an earlier stage of the tradition behind Diophantos). Links to the theory of figurate numbers are also suggested, and pebble-hence to a abacus-representation of the naïve-geometric procedures (cf. above, the end of chapter VI).

Another possible line of transmission of Babylonian influence goes to the pre-Diophantine algebraic tradition. I have already pointed out the similar ways in which the Babylonians and Diophantos deal with non-normalized problems, and other similarities could be found in that tin part of Diophantos' Arithmetica which possesses cuneiform parallels. Such similarities are, however, fairly inconclusive, since the subject-matter itself restricts the range of possible procedures strongly. Supplementary evidence may, however, be hidden in a much-discussed term of the Arithmetica, the πλασματικός, which occurs in i.xvii, i.xviii and i.xxx of the surviving Greek part, and in the Arabic IV.17, V.19 and V.7. In diorism, i.e. the Greek text, it seems to be the condition for solvability which is called πλασματικόν, while the Arabic passages speak of the whole problem as belonging to the class of al-muḥayya'ah.

The Greek term derives from πλάσμα, "to form", "to mold", etc., and it is related to πλασμος, "anything formed or molded, image, figure" etc. (GEL 1412a). Because of this etymology and the Greek passages alone, Ver Eecke suggested it to mean that the diorism can be demonstrated geometrically. Since a reference to Euclidean geometry fits badly to the distribution of the term.
in the Arabic books, both editors of the Arabic text have looked for alternative ways to get a meaning of the term in its actual contexts (191). Here again, however, the naive-geometric view-point changes the basis of the question. We already know a māgūm, a fixed figure or "mold" on which the diorisms of the three Greek passages can be seen immediately -- viz., the upper square in Figure 20 (quartered as in Figure 17, since Diophantos uses semi-sums and semi-differences). Moreover, the diorism of the Arabic IV.7 can be seen on the three-dimensional analogon of the same figure.

The diorisms of the Arabic IV.17 and IV.19 are of a different character, involving factorizations of the sides of cubes. There are no direct links to specific Babylonian material; on the other hand, certain techniques used for the computation of large reciprocal tables and the techniques of scaling are akin to the Diophantine procedure. Since (at least the Arabic) text does not claim that these and none but these problems possess a distinctive mathematical quality but only states that they belong to a certain pre-established bunch of problems possessing the quality, we should perhaps interpret the term as designating problems the feasibility of which is seen by certain naive-geometric procedures (not necessarily by Diophantos but at least by the people who established the bunch).

The interpretation is not compelling, nor is however any rival explanation. A hint of a Babylonian connection may -- but need not -- hide behind the term and the concept.
X.4. A direct descendant: *Liber mensurationum*

If the inspirations from Babylonian algebra to Greek mathematics can only be traced indirectly, through the combination of many sorts of indirect evidence, influences in Medieval Islamic mathematics are direct and easily verified.

Once more, I shall only sketch the basis of the argument, since I deal with the matter in detail elsewhere. The central source is a Latin translation made by Gherardo di Cremona in the 12th century from an Arabic original due to one (otherwise unidentified) Abū Bakr, the *Liber mensurationum*. The first parts of the work deal with squares and rectangles (the later parts, related to Alexandrinian practical geometry, does not concern us here). It was already noticed by H.L. Boursin in his edition that the work shares many problem-types and even the coefficients of certain problems with Babylonian algebra (making no distinction between Old Babylonian and Seleucid material). This, however, is not conclusive. Starting from the simplest cases you will necessarily hit upon many of the same problem-types -- and if you prefer the second-simplest to the simplest Pythagorean triangle, your numbers will be 6, 8 and 10.

The first decisive observation is that many problems are solved twice, first by a method given no specific name (and hence to be regarded as the normal, fundamental method) and next by *algebra*, obviously a term meant to render the Arabic *al-jabr*. In a general sense of the word, both methods are equally algebraic. *Algebra*, however, refers directly to the fundamental cases known from al-Khwārizmī. It is hence the rhetorical discipline known from al-Khwārizmī and ibn Turk and also referred to by Thābit ibn Qurra in his "Rectification of the cases of *al-jabr*".
In several cases, the numerical steps of the fundamental method and the alternative by al-Jabr are identical. The difference between the two must therefore be one of representation and conceptualization.

The next observation is that the discursive organization of the descriptions of the "fundamental procedures coincides down to the use of choice of grammatical tense and person and to certain standard phrases ("since he has said"; "may your memory retain") with the familiar structure of Old Babylonian texts. The procedures are also often those known from the Old Babylonian texts — e.g. the "change of variable" of AO 8862 No 1. The standard length-width-problem is solved by means of semi-sum and semi-difference, showing that the connection of the text is really directly to the Old Babylonian tradition, bypassing the Seleucid astronomical school.

A closer look at the vocabulary shows that the conceptual distinctions known from the classical Old Babylonian tradition are not respected completely; so much remains, however, that we have good reasons to believe that a naive-geometric method is behind the numerical algorithms described in the text. A final "See" after many procedure-descriptions indicates that the original has indeed contained (naive-)geometric justifications of the methods.

These observations are the main but not the sole reasons to see the fundamental approach of the text as a direct continuation of an Old Babylonian naive-geometric tradition, which must then have been alive until the Arabic original was written (probably not much later than A.D. 800). Even in Abū Kāmil's Algebra, dating from c. A.D. 900, an alternative to the normal al-jabr procedure is sometimes offered, which contains the typical Old Babylonian steps, though in arithmetico-rhetorical
disguise). More striking is, however, a passage in Abū'l-Wafā's Book on What is Necessary from Geometric Construction for the Artisan, written after A.D. 990. In chapter 10, prop. 13, the author tells that he has taken part in certain discussions between "artisans" and "geometers", apparently regarded as coherent groups. Confronted with the problem of adding three (equal) geometric squares (the sum also being a square), the artisans proposed a number of solutions, "to some of which were given proofs",--proofs which turn out to be of cut-and-paste character. The geometers too had provided a solution (in Greek style), but that was not acceptable to the artisans, who claimed a concrete rearrangement of parts into which the original squares could be cut.

In chapter I, al-Khwārizmī's naive-geometric justifications of his algorithms were explained as a pedagogical device, in order to demonstrate what naive geometry would look like. At the present stage of the investigation it turns out that the Old naive-geometric tradition was still alive when al-Khwārizmī wrote his seminal compendium on algebra. We can hardly assume that he invented anew a technique which was widely practiced around him—and we can therefore be confident that his justifications were direct descendants of those of the Old Babylonian calculators. We may guess that even his arithmetico-rhetorical al-jabr derives ultimately though highly transformed from the same source, but there we have no direct evidence; through his justifications, however, we know that the Ancient techniques were passed on to Medieval Islam and to the early European Renaissance, and hence to the modern world.
1. The Old Babylonian period spans the time from c. 2000 B.C. to 1600 B.C. (middle chronology). The mathematical texts dealt with in this paper belong (with the exception of the Seleucid text presented first) to the time from c. 1800 B.C. to c. 1600 B.C.

2. Anachronisms are lurking everywhere when one speaks of Babylonian mathematics in modern terms. The Babylonians did not classify their problems according to degree -- they have related classifications, but the delimitations deviate somewhat from ours, and they have another basis. "Equations", on the other hand, is a fully adequate description even of the Old Babylonian pattern of thought, if only we remember that what is equated is not pure number but the entity and its measuring number: Combinations of unknown quantities equal given numbers or, in certain cases, other combinations of unknown quantities.

3. BM 34560 No 9 (BM 34560 refers to the museum signature, No 9 to the number of the problem inside the tablet as numbered in the edition of the text). The text was published, transliterated, translated and discussed by Neugebauer in HKI III, 15ff. The numbers in the margin refer to the position of the text on the tablet: Obverse/reverse, column No, line No.

The text is Seleucid, i.e. from around the 3d century B.C.

The translation is a literal retranslation of Neugebauer's German translation as given in HKI III. So, it renders the way in which Babylonian algebra is known to broader circles of historians of mathematics. (All translations given below will be my own direct translations from the original language).

4. For the transcription of the sexagesimal place value numbers found in the text I follow Neugebauer's system, which in my opinion is better suited than Neugebauer's for the purpose of the present investigation: 3'' is the same as 3, 3' the same as 3·60^{-1}, 3'' means 3·60^{-2}, etc. 3' means 3·60^{1}, 3'' equals 3·60^{2}, etc. The notation is an extension of our current degree-minute-second-notation, which anyhow descends directly from the Babylonian place value system.

I use the notation as a compromise between two requirements: For the convenience of the reader, the translations must indicate absolute place; this is not done in the original cuneiform, but
so few errors are made during additive operations that the Babylonians must have possessed some means to keep track of orders of magnitude. On the other hand, the zeroes necessary in the conventional transcription introduced by Neugebauer (1932) (3,0;5 instead of Thureau-Dangin's 3.5' and the Babylonian 3 5') are best avoided in an investigation of Babylonian patterns of thought, where such zeroes had no existence. (Actually, the situation is different in an investigation of mathematical techniques, especially the techniques of mathematical astronomy, with special regard to which Neugebauer introduced his notation).


6. Arithmetica I, xxvii.

7. The term is due to Nesselmann (1842:302ff), who also introduced the more current "rhetorical algebra".

8. Irrespective of the question whether "geometric algebra" was or was intended to be an "algebra".

9. Cf. also Elements II,5. An analogue of the algebraic problem in one unknown is found in Data, prop. 50, and in Elements VI,28.

10. In a preliminary discussion paper (Heyrup 1985) I spoke of "geometrical heuristics". I have also pondered "visual" or "intuitive geometry". After much reflection, however, I have come to prefer "naive geometry" as relatively unloaded with psychological and philosophical connotations.

11. See Rosen 1831:15-16.

12. The immediate argument for this is that symbolic algebra requires a level of abstraction which appears to be totally alien to Babylonian thought. If this seems too much of an argument ex silentio, it can be added that symbolic algebra is grosso modo akin in structure to arithmetico-rhetorical algebra. So, even if we upkeep the possibility of symbolic algebra as a silent hypothesis, the arguments which will be given later against an arithmetico-rhetorical interpretation will also exclude symbolic translations of the latter.
On the same account, an "abacus" representation of Babylonian algebra (with counters representing the coefficients of the products and powers of the unknowns) can be discarded. In itself, the "abacus interpretation" might have a certain plausibility, since material calculi had been used for common reckoning and/or computation in earlier epochs in Mesopotamia. Nothing, however, but the writing material (pebbles instead of ink) distinguishes such a representation from the syncopated algebra of Diophantos or the further development and schematization of the same principle found in Medieval Indian algebra; arguments against an arithmetico-rhetorical interpretation of Babylonian algebra will hence also be arguments against an arithmetical "abacus algebra" (I shall return below to the possibility of a geometric "abacus algebra" related to the Greek "figurate numbers").

13. By "Modern" I mean "post-Renaissance", in the case of algebra specifically "post-Vieta". I disregard what mathematicians would call "modern" (abstract, "post-Noether") algebra as irrelevant to the present discussion: It is, at least in classical senses of those words, neither arithmetical nor geometric, be it in basic conceptualization or in method (although it is, primarily, an abstract extrapolation from arithmetical conceptualization and method).

14. It should be emphasized that the investigation deals only with the algebraic texts. There is no reason to doubt the purely numerical character of many of the table texts; but the numerical character of texts like Plimpton 322 (MCl, 30) does not permit us to conclude that algebraic problems too were understood and solved arithmetically. Similarly, it cannot be doubted that a number of texts deal with real geometric problems, but even there generalizations are not automatically justified.

15. Among the most explicit, Ihureau-Dangin (1940:302) states that the problems dealing with geometrical figures do so because "a plane figure will easily give rise to a second-degree equation", but that the problems are still "purely numerical", just like the indeterminate equations of Diophantos' *Arithmetica* VI, for which right triangles function as a pretext.

16. So, van der Waerden (1961:71ff) suggests hypothetically that certain basic algebraic identities may have been proved geometrically
(\(a-b\))(a+b) = a^2-b^2, etc.). The conjecture is accepted by Vajman (1961:168f). At the same time, however, van der Waerden distinguishes the method of proof from the conceptualization, stating that the "thought processes of the Babylonians were chiefly algebraic (i.e., arithmetico-algebraic -- JM). It is true that they illustrated unknown numbers by means of lines and areas, but they always remained numbers".

17. IH 52301, the inscription on the edge as interpreted by Bruins (1953:242ff, 252).


20. To, no real Sumerian dictionary exists to this day.

21. The prevailing tendency has been to leave the conception of ideograms and to claim that the cuneiform signs when not used phonetically would stand for, and be read as, specific Akkadian words. The difference between an ideogram and a logogram is the difference between "a" and "\textit{vizz}". The first sign will of course always be read by words, depending on the situation as "plus", "added to", "and", or something similar; only in the specific additive meaning, however, can it replace the spoken word "and" -- it is no logogram, it corresponds to an operational concept which is not identical with any verbal description. "\textit{Vizz}" on the other hand, is a real logogram for "namely".

No doubt, the logographic interpretation describes the normal non-phonetic use of cuneiform signs adequately. At least in mathematical texts, however, certain signs must be understood as ideograms, not as logograms, as I shall exemplify below (cf. notes 57-58; and note (4) to TMS XVI A; cf. also 51a, 25f, on similar phenomena in non-mathematical contexts).

22. The sign \(\square\) may be taken as an example. The conventional sign name is KAŠ (the name given to it in Ancient sign lists). It may stand for Sumerian kaš, "beer" (Sumerian words are usually transliterated in spaced types), and for the possessive suffix -bi; the latter reading is used in Sumerian as an approximate syllabic writing for the compound bē > bē, "says it" (or rather "it is said"). These three uses have given rise, respectively, to logographic use in
Akkadian texts for the corresponding words Sîkarsû, *šu*-ša and ḍabûm, together with the derived ẞ0/ẞatu, "this" (a function in which Sumerian bi can also be used). In the Old Babylonian period it will also be found with the phonetic values bi and bé and, more rarely, pî and pê (accents and subscript numbers are used to distinguish different writings of the same syllable). In later periods, it can also be used phonetically as gê, koê and kôê.

To this comes the role in a number of composite sign groups used logographically: different specified sorts of beer; innkeeper; song(?); etc. Finally, the sign may represent twice the surface unit ßêê, written —— (). (After HeA and ABz, No 214), and a commentary from Bendt Alster).

23. To know whether he thinks concretely through the standard-terminology we would have to investigate whether he avoids using it when constructing the orthogonal to a non-horizontal line; i.e., we would have to investigate the structure of his total terminology and its use in various situations.

24. See Pauly-Missawa, 5. LAHw quotes the Semitic root in Hebrew, Arabic and Aramaic. It appears to be absent in Akkadian.

25. The Semitic languages combine (with special clarity and richness in the system of verbs and their derivations) fixed, mainly consonantal roots carrying the semantic basis, with a huge variety of prefixes, infixes (among which the vowels, which are submitted to change) and suffixes determining not only grammatical category but also many semantic displacements which in Indo-European languages are not subject to morphological regularity. The actual functioning of such a system requires that its speakers apprehend (subconsciously) all the derivations of a root as belonging to one scheme, in the way an English four-year-old child apprehends "whistled" as a temporal displacement of the semantic basis "whistle" according to a general scheme (as revealed by its construction of forms like "goed" instead of "went").

26. VAT 8390, rev. 21 (MKT I, 337).

27. The former interpretation is suggested by the use of Sumerian ku, "to eat", as a logogram for the term (cf. below, section IV.3). For this reason, it is normally accepted today, cf. von Soden 1964:50, and Lahw, kullu(m) and škēlu(m).

The latter interpretation was proposed by Thureau-Dangin (e.g. TMB, 219f), who explained the logographic use of ku as a pun-
like transfer, inspired by the coincident St-forms of kūllum and skāšum (cf. IN5, 232f). Such transfers are indeed not uncommon in cuneiform writing (cf. above, note 22), and hence a derivation from "holding" cannot be outlawed. As it will appear below, a relation to another term (takiltum) appears to rule out the derivation from "eating"; the connection to "holding", on the other hand, will turn out to make perfect sense.

28. A simple instance of such structural analysis was suggested in note 27 as a means to investigate whether a modern user of geometrical terminology associates the "raising" of a perpendicular with the literal meaning of this term.

29. This paradoxical phrase should perhaps be clarified. An important characteristic of a technical term is fixed semantic contents and relative absence of connotations and analogic meanings. Technical terms when applied as such are not open-ended. Even in modern mathematics, however, technical terms are also used metaphorically and in other ways departing from their technical semantics. This happens during theoretical innovation, when the technical terminology has to adapt to new conceptual structures; it also occurs in informal discussion and didactical explanation when the truth is not to be stated but to be discovered or conveyed. These are processes which always require compromise with pre-existent understanding, and therefore such non-technical displacements of meaning reveal something about this understanding. (Cf. for certain aspects of this discussion B.F. Beck (1978) and S. Marcus (1980)).

The Babylonian mathematical texts abound in examples of such derived meanings and applications of terms -- to an extent which suggests that we are not confronted with a real technical terminology after all, that few terms possess a basic, really fixed technical meaning. Instead, most terms should probably be regarded as open-ended expressions which in certain standardized situations are used in a standardized way. This will be amply exemplified below.

30. This is, grosso modo, the way I go through the subject in my preliminary presentation (Heyrup 1985) of the problem and of my results. The outcome is rather opaque.
31. With the partial exception of *esegum* (and its logographic equivalent *tab*), the original meaning of which is "to double", and which in phrases "double x to n" means "multiply x by [the positive integer] n" if interpreted arithmetically.

32. It should, however, be emphasized that both Neugebauer and Thureau-Dangin show great intuitive sensitivity to the shades of the vocabulary in MK1 and TM1. I remember no single restitution of a broken text in either of the two collections which does not fit the results of my structural investigation.

33. Disregarding the possibility to distinguish between multiplications involving only integers, multiplications where one factor at least is an integer, and multiplications of wider classes of numbers. In fact, all Babylonian terms except *esegum* (and *tab*) can be applied for the "multiplication" of any number by any other number.

34. The vocabulary of the later (Seleucid) mathematical texts is very different, and can indeed be taken as an indication that the mathematical conceptualizations had changed through and through during the centuries which separate the two periods. Cf. below, section X.2.

35. In order to emphasize the purely Old Babylonian character of the summary I write all Akkadian verbs and nouns with "nismation", i.e. with the final -n which was lost in later centuries.

36. Literally, the Sumerian *g₃₃₃₃* means something like "to lay down (g₃₃₃₃) repeatedly"; possibly, the UL of UL.GAR is due to a sound shift from UR₃₃₃₃, *inter alia* "to collect" (SL 11, No 575.9), which would lead to an interpretation of UL.GAR as a composite verb "to lay down collectedly" (maybe an artificial "pseudo-Sumerogram").

37. Cf. section VIII.2, the notes to AO 0062, for reasons why the single sum has to be understood as a plural.

38. A similar use of Akkadian *al₃₃₃₃*, "to go", as a substitute for *esegum*, "to double", is found in several Suse texts (among which TMS IX, translated below in section VIII.3).
39. Originally, Thureau-Dangin suggested the conjecture that
nīm might be used for the factitive or causative S-stem
nīm (THB, 239). However, the headline of the Suss list of con-
stant (i-gi-gub-) factors claims to contain "i-gi-gub, that
of making anything high" (THS III, 1), using the infinitive yullûm
of the constantly factitive D-stem. Since the S-stem is furthermore
used (in AO 17264, MKT I, 126f, and in Haddad 104, III, 7, al-Rawi
& Roaf 1985) in the sense of making a square-root "come up" as
a result, nīm-yullûm is probably to replace Thureau-Dangin's
conjecture.

40. As we shall see below, the area of a rectangle is presumably
also found by "raising", although the operation is normally not
made explicit.

41. BM 85196, rev. II, 11 (MKT II, 46).

42. Gandz 1939:417f

43. The same idea of covering a piece of land is indeed seen in the Old Baby-
lonian measurement of a slope by the "yukullûm eaten in 1 cubit", i.e. cover-

44. YBC 4675, obv. 1 (MCT, 44) has the expression "Šuma ašā
ušī sī-ku", "when a length and a length est/hold a surface", re-
turning to a surface stretched by two (different) lengths
(i.e. to an irregular quadrangular surface). Later in the same
text (rev. 15) the term Sutakkulûm itself stands as a complete
parallel to the use (in rev. 6) of ṣūbrum, "to make", "to produce"
(viz., a quadrangular surface). In neither case is any multipli-
cation to be found.

45. On the denotation of squares, see Deimel 1923 № 82 (cf. MKT
I, 91, and Powell 1976:430) and Edzard 1969. On the equality of
lengths alone or widths alone, see Allotte de la Fuâe 1915:137ff.

46. Occasionally ba-si. This term is, however, more common
in connection with cube roots.

47. BM 15285, passim (MKT I, 137ff). The geometrical character
of the squares is certain both because they are spoken of as po-
sitioned and because they are drawn on the tablet. Shifts between
the two terms show that šub-ši, intended here as a logogram
for the Akkadian word mithūrum (cf. immediately below). In the
"algebraic" problem text Str. 363 (MKT I, 244), where the scribe


Has done his best to find (and, one may suspect, to construct) Sumerian logograms to express his Akkadian thought, the same equivalence (b-ṣi,-mûḫarûm) is used.

48. Private communication.

49. See e.g. BM 13901, passim (several problems are translated below).

50. AO 17264, obv. 2–3 (HKT I, 126).

51. Goetze 1951.

52. Texte V, THS 35ff. All three occurrences are late Old Babylonian, AO 17264 possibly even early Kassite.

53. The sign is indeed a "squared line": . So, it is unclear whether its ideographic value should be connected to its use as a logogram for lowûm, "to surround", or should be considered iconic. (In both cases, the use in AO 17264 must be considered secondary, derived from the habitual association of the quadratic figure with equality).

54. Texte VI, THS 49ff.

55. BM 85194 (HKT I, 143ff) and BM 85196 (HKT II, 43ff).

56. Texte IX, 5 and 12, and Texte XXI, 4 (THS, 63 and 108). The edition transcribes as Šutûhûrum and translates as Šutûkûrum.

57. Cf. above, note 21. The ideographic role of the sign in connection with squaring and "rectangularization" should of course be distinguished from its possibly logographic role inside other semantic fields.

The sign is , a repeated LAGAB. As in the logogram l-ki-ki, the repetition looks like an intentional graphic representation of the reciprocity of the št-atom Šutûkûrum and Šutûhûrum, or perhaps of the use of two lines to stretch the square or rectangle. Cf. also note 58 on UL.UL and UR.UR.

58. YBC 4662 and 4663, passim (MC1, 69, 71f). In YBC 4662, the term occurs in the construction "x à-rá x UR.UR.a"; however, in several other constructions (appending, i.e. an additive operation; raising) the tablet writes a-rá instead of ana, due perhaps to a dictation or writing error; so, I guess that the original was "x ana x ...". In YBC 4663, the term when used for squaring gives the factor only once ("3’15’ UR.UR.ta"), but for once Šutûkûrum
is used in the same way in that tablet (rev. 20). On the other
hand, while the tablet has "u $ s a g UR.UR.ta" (t a - i n, "from/
"by means of"), it writes "u $ u s a g B u t ü k i l l" (u - "and"); UR.UR
can therefore not be a pure logogram for B u t ü k i l l; instead the
whole phrase is written as an ideographic syncope.

Goetze (MCT, 148) counts the two tablets among the early Southern
ones. Both, however, state results with the word t a m m a r, "you
see", as do the texts belonging to his group VI and other Northern
texts (cf. below, note 84).

As in the case of i - k ū - k ū as a logogram for B u t ü k i l l, the
repetitive structure of U R.UR is probably to be read as a (pseudo-)
Sumerian rendition of the reciprocity of the Št-form B u t a m h ü r u m
-- or, rather, as a way to render in Sumerian grammar a geometrical
idea which is rendered in Akkadian by the Št-stem (and rendered
badly so, as the verb has only one object).


60. All three values appear to belong originally to U R, but all
are also testified for U R - cf. the terms in question in A H w, and
M E A, No 401 (U R), and No 575 (U R).

61. Str. 363, passim (MKT I, 240f); Str. 368, rev. 5, 8 (MKT I,
311); VAT 7532, obv. 19 (MKT I, 295); VAT 7535, rev. 17 (MKT I,
305); VAT 7620, passim (MKT I, 315); Y D C 6504, passim (MKT III,
22f).

62. Str. 363, rev. 15-16: "... 20 y 1 UL.UL-ma 20 / 40 y 5 UL.UL-
ma 320"...". Furthermore, in obv. 9 of the same tablet a relative
clause refers back to U L . U L by a syllabic B u t ü k i l l.

63. Y D C 6504. In the first two problems of the tablet, Š b - Š i l ,
is used in the statement, while B u t a m h ü r u m is used for squarings
in the prescription of the procedure, and Š b - Š i l turns up when
towards the end a square-root is taken. In the third and fourth
problems, U L . U L is used both in statement and procedure for
squarings, while Š b - Š i l is still used for the square-root.

64. Falkenstein 1959:20 (§ 9,1.b, "t"); SL a 45, § 28, cf. § 30.

65. A place where the distinction between "halving" and "division
by 2" (i.e. multiplication by 2) is especially obvious is Str.
367, rev. 3-4 (MKT I, 260). A clear distinction between Š ŋ ŭ t ŭ m
and Š ŋ ŭ t ŭ m is found in the tablets A O 8862 (below, section XIII2)
and EM 13901 (MKT III,1-5). A single tablet (Y D C 6504, MKT III,
22f) uses Š ŋ ŭ r ŭ s where others have Š ŋ ŭ t ŭ m.
66. The standard table of reciprocals lists the reciprocals of the regular numbers from 1 to 1'21' (c.01) -- cf. MKT I, 9ff. It can be legitimately discussed whether our term "table of reciprocals" is anachronistic. Indeed, one table, which appears to antedate 1850 B.C. (MKT I, 10 No 4), seems to express the idea that not 1/n but 60/n is tabulated (Scheil 1915:196). According to Steinkeller (1979:187), another table with phonetically written numbers suggests the same idea (in MKT I, 26ff). On the other hand, such conceptualizations of early tables have no necessary implications for the understanding which Old Babylonian calculators had of the tables used in their own times, and two observations combined suggest that they did in fact apprehend their own tables as tabulations of the numbers 1/n. Firstly, they used the tables for divisions, i.e. for multiplications with these numbers. Secondly, there is textual evidence that they possessed a specific concept for the number 1/n, as distinct from a general "n'th part" (of something) -- cf. below, note 69.

67. A few tables containing approximate reciprocals of certain irregular numbers exist: YBC 10529 lists reciprocals of regular as well as irregular numbers between 56 and 1'20' (MCT, 16). N 10, John F. Lewis Collection, Free Libr. Philadelphia gives reciprocals of 7, 11, 13, 14 and 17 (Sachs 1952, 152). Apparently, however, such approximations are not used in the Old Babylonian mathematical texts (and since the irregular divisors of these texts always divide the dividends, such use would indeed lead to errors which could not go unnoticed).

68. VAT 4768 and VAT 4675, published by Förster (1976 No 65 and 175), transliterated and transcribed by Bauer (1967:500-11). The texts belong to the fourth year of Lagelanda, and speak of 1/4 Šekel silver and 1/6 Šekel silver, by the phrase "lígi₄ n g₅₂₃₁". Similar contemporary evidence (also from Lagal) is found in M. Lambert 1953:60,102,105,106,108,110 (1/3, 1/4 and 1/6 Šekel of silver or lead) and Alliotte de la Fuye 1915:132 (1/4 šar of lamb).

All these tablets antedate the first known occurrences of sexagesimal reciprocals by some 350 years, and they antedate by c. 200 years a school text which suggests that the ideas behind the sexagesimal system were on their way but not yet mature nor formulated around 2200 B.C. (Limet 1973 No 36; cf. commentaries
in Powell 1976:426f and Heyrup 1982:28). We can therefore confidently infer that the general sense of a reciprocal is a secondary derivation. This undermines the only plausible (yet grammatically somewhat enigmatic) explanation of the term given to date, one offered by Bruins (e.g. 1971:240): literally, the phrase "i gi 6 gāl-bi 70-ānu" could mean "in the front of 6 is: 70 is it", i.e. "in front of 6 is found what (in the table of reciprocals?) 70". This explanation would interchange basic and derived meaning, and until evidence turns up which moves the tables of reciprocals back into the mid-third millennium, it cannot be upheld.

Truly, Bruins (1983:105, and earlier) points to two Old Babylonian texts which write the Akkadian term pānu, "in front of", when wanting to designate the reciprocal. (So does also Haddad 104, see al-Rawi & Rosaf 1985, section 0.4.3). Certain Old Babylonian scribes hence appear to have held the same hypothesis as Bruins concerning the origin of the expression. But Old Babylonian scribes may as easily have constructed a scholarly pseudo-etymology as they can have guessed correctly a conceptual development which had taken place some 800 years before their own time. In any case, current logographich use of i gi for pānum may easily have led them astray to an erroneous "folk etymology".

69. Str. 367 (HK I 259f) speaks in obv. 3 of "the third part" of a length in a complete phrase "i gi 6 gāl", while the reciprocals of 4, 1, 1, 3, 20" and 1'12' are spoken of (passim) simply as "i gi n". The same distinction is made in VAT 7532 and VAT 7533 (HK I 294ff and 303ff); here, even the n'th part of the number 6 is spoken of in the complete phrase when this number 6 is taken to represent an unknown length, and the part hence understood as a fraction of something, not as a reciprocal (a number). In BM 85210 rev. 1, 0-22 (HK I 221f), the "n'th part of n" is also spoken of by the complete expression and the reciprocals simply by "i gi n"; but furthermore, the finding of the latter is spoken of by the usual term du₃ (-patārum, "to detach", cf. below), the process leading to the former is designated by zi (-naggārum, "to tear out"). BM 85194 (rev. 1,28, rev. III,2,3, and passim; HK I 143ff) speaks of both "part" and "reciprocal" by means of the abbreviated expression, but distinguishes by means of the differentiation between zi and du₃.

70. Thureau-Dangin 1936:56.
71. In Str. 367 (AHK I, 259f) a triangle of area 21'36' is "detached" from a trapezium of area 36', leaving a rectangle of area 14'24'. The other subtractive occurrence is Str. 362, obv. 15 (AHK I, 240).

72. Cf. also the subtractive conceptualization of the process "to find the n'th part of m" in BM 85210 and BM 85194 (see note 69).

Further evidence against Thureau-Dangin's assumption comes from the way the finding of a square-root is spoken of: You are requested to "make the equilateral come up" (šološelom); you "take" it (šaqum); or the question is asked, "what the equilateral" (minšī b-i sī). Had pāšum meant simply "to solve" an arithmetical problem, nothing would have prevented the Babylonians from using it also for the solution of the problem x·x = A.

73. VAT 8391, rev. I.28-30 (below, section VII.2); VAT 8512, rev. 1-5 (AHK I, 341); VAT 8520, obv. 24f, rev. 25f (AHK I, 346f); Str. 363, passim (AHK I, 244f).

74. Str. 367, rev. 11 (AHK I, 260); VAT 8512, obv. 10-12 (AHK I, 341). A possible exception is AO 6770, No 1, lines 5-7. Still, since no really satisfactory interpretation of this text has been given, it can hardly serve as evidence for anything. (Improved transliteration and bibliography of earlier treatments of this text will be found in Brennjes & Müller 1982; cf. Høyrup 1984 for reasons why even this latest interpretation is problematic).

75. Strictly speaking, the Akkadian terms are not just rare. In reality they are never used as names for the standard variables but only in a couple of texts dealing with real rectangles: Ub., 146, obv. 3 (in Baggir 1962, Pl. 3; Siddow alone and IM 53965, passim (in Baggir 1951; both Terres). On the use of pāšum (plural of pūšum) to designate the sides of a real square in BM 13901, No 23, cf. below, section V.4. (Three final instances deal with carrying distances for bricks and the width of a canal).

76. See the texts from c. 2400 B.C. published and discussed by Allotte de la Fuücke (1915). A difference between the Early Dynastic surveying texts and the Old Babylonian standard algebra problems should be noted: While the latter tell us that they deal with a rectangle simply by speaking of u ā and sāg without
any epithet (saying thereby implicitly that there is only one length and one width), the former will normally present all four sides of a quadrangle, and if a pair of opposing sides are equal they will (with one exception which seems most hastily written) tell explicitly that this is $u\,\text{si}_1$, "lengths being equal", or $s\,\text{ag}_1$, "widths being equal".

77. THS V and VI. Even if the length is spoken of explicitly, the same lines of the texts will also treat the square figure itself as a number, viz. as the same number as the "length". Here as everywhere, square figure and side are conceptually conflated. So THS V, obv. 11-13: "the square line and $1/11$ of its length accumulated: 1", i.e. square-line $\times$ length $= 55$.

78. On the other hand, the terms $u\,\text{s}$ and $s\,\text{ag}$ are (on the same and other sorts of evidence) not real logograms -- cf. below.

79. Like $u\,\text{s}$ and $s\,\text{ag}$, a-šā is used already in Early Dynastic texts (cf. note 72). It seems plausible that this rooting in an old tradition should be linked causally with the all-dominating Sumerographic writing (in fact, full phonetic writing of $c\,\text{jum}$ is as absent as phonetic/writing of $u\,\text{s}$ and $s\,\text{ag}$). In contrast, the unknown "squared line" in problems of one unknown is never written by the traditional Sumerogram $\text{si}_1$, (cf. note 45). This appears to indicate that theoretical algebraic problems (among which the problems of one unknown are important) did not arise until the Old Babylonian age (or that they arose among Akkadian speakers -- in which connection it may be of interest that a specific Akkadian record-keeping system, distinct from the contemporary Sumerian system, was in use during the Sargonic era, see Foster 1982:22-25). A similar conclusion could be drawn from the greater part of the basic algebraic vocabulary, which is written alternatingly in phonetic and ideographic writing, but where the latter writing is not traditional Sumerian.


81. More complete information on the Old Babylonian metrological system will be found in THB (pp. xiii-xvii) and MCT (pp. 4-6).

82. YBC 6504; panām (HKT III, 22f). In the same text, intermediate results are "posed".
83. IM 52301, obv. 19f (below, section X.1); the text is rather late and contains several other deviations from normal usage); IM 54470, obv. 7 (Baqir 1951:30). In the newly discovered text from Tell Haddad (Haddad 104, IV.9,17,29; in al-Rawi & Roaf 1985) the form luwrap (D-stem, stative) is used of numbers which "stand written down" in a table of constant factors.

84. VAI 8520, obv. 21, rev. 20 (HKT I, 346f); YBC 6967, obv. 11. Cf. below, sections V.1 and VIII.4. A slightly different phrasing is found in IM 52301, rev. 5 and 10 (cf. note 79) and in Db,146, 4 and 13 (Baqir 1962:P1. 3), and another possibly in TMS XVII, 12.

85. "Posing" stands precisely as nadum in TMS XIII, 10 (cf. correction to the line in von Soden 1964:49) and in IM 53965, rev. 7 (Baqir 1951:39). In AO 0862, 11, 21f (HKT I, 110); BM 13901, obv. 11. B (HKT III, 2), YBC 4662, obv. 21 and 33 (HKT,71), and finally in YBC 4663, rev. 23 (HKT, 69), the "equilateral" is inscribed until twice.

86. Most recent edition with addition of a large fragment in Saggs 1960.

87. Allw, article "nadum III", §§ 20, 22, 24.

88. sum and nadum are found in the texts to which Goetze ascribes for linguistic reasons an early, southern origin (groups I-IV, see HKT, 146-151). Ilium is found in his group VI ("northern modernizations of southern (Larsa) originals"), in the Susa texts of TMS and in a number of the late (and northern) Tell Ujarma texts (in Baqir 1950a and 1951); the early Tell Ujarma text IM 55357 (Baqir 1950:41-43) uses iqi-d³, a logogram for laqar, mistook by homophony for iqi-du, which is used in the same function in YBC 4669 (rev. 1, 5-7; HKT III,27) and YBC 4673 (rev. 11, possib. HKT III,31); these too are probably northern, cf. HKT I, 387f and 123fr. iji, iji, iji and related derivations from elum are found in Goetze's group V ("northern characteristics", maybe somewhat older than the group VI texts); in the remaining late Tell Ujarma texts (Baqir 1951); and finally in the early northern texts Db,146 (Baqir 1962:P1. 3) and Haddad 104 (al-Rawi & Roaf 1985).

Only very few exceptions to these clear-cut rules are found. The group I text YBC 7997 (HKT, 90) aligns nadum and elum (the former being used for final results alone); another group I text
(YBC 4675, with the parallel fragment YBC 9852 -- 467, 44f) uses glūm exclusively. lammar is used alongside with nadānum in YBC 4662, which Goetze locates in his group II (Larsa?), and it is used alone in MLC 1950 (MCT, 48), which shares a specific Sumerian standard phrase with a number of texts belonging to group III but is otherwise unlocated. Finally, lammar and glūm are found together in the late Tell Šarq钻 texts (IM 54339 — Beqir 1951:41), while igī alone is found in VAT 672 (MKT I, 267), a fragment with other stylistic peculiarities and containing too little Akkadian to allow for linguistic analysis.

89. MKT I, vll. MKT III, 5 continues "Wer Terminologiegeschicht- liche Studien an Hand einer Übersetzung machen will, dem ist doch nicht zu helfen".

90. So, ēpešum and the logogram ki when used as verbs are rendered "to make"; the infinitives used as nouns by "the making"; and nēpešum by "the having-been-made". An instance of enforced deviation is the complex naḫārum/Nutamūrum/mithurtum/mehrum, rendered by "correspond to"/"raise against each other"/"squared line"/"counterpart" (cf. section III.4).

91. So, in a genitive construction like "ib-si, 15"", the preposition "of" is given in normal writing, "the equilateral of 15". "mišil u 8" will be translated "half of the length", because the construct state mišil indicates a genitive construction, although no genitive marker is joined to u 8.

92. I discuss the problems of two-dimensional geometrical conceptualizations and methods and a number of complex algebra problems in my preliminary (1985:41-63, 105.1-105.42).

93. Another text dealing with igūm and igabum is VAT 8520 (MKT I, 346f). There, the names of the two unknowns are written syllabically throughout the tablet, while "part" and "reciprocal" are referred to by the usual ideogram igī. This leaves little doubt that the two ideas were, and thus have to be, kept apart.

94. In various problems from BM 13901 (below), the supplementary square is appended to the gnomon; in VAT 8520, as in the present text, the gnomon is appended.
95. The one exception from this general rule is IM 52301 (obv. 12, rev. 10). This is only one of several reasons to regard this late tablet as a symptom of changing conceptualizations -- cf. below, note 113; section X.1; and note 176.

96. In its own way, this confirms Neugebauer's old intuition. Thureau-Dangin suggested very tentatively (1936a:31 n.1) that wäștum might designate absolute unity (as distinct from 1', 1 etc.). Against this, Neugebauer (HKI III, 11) raised the objection that only absolute unities belonging with problems of one unknown were designated wäștum. Instead, he suggested that the term might designate a certain class of coefficients of value 1. Irrespective of the precise interpretation, indeed, the "projection" is a coefficient 1 of dimension [length], multiplication by which transforms a linear quantity into a quantity of dimension [length'].

97. In YBC 6967, this quantity was spoken of by the noun tekitum, here however by the relative clause "which you have made span", te tuštakkijû. This parallel (which is repeated copiously) confirms the close relation between "making span" (atuškium) and tekitum.

98. Thureau-Dangin 1936a:27; HKI III, 10. The criteria are language and writing.


100. HKI III, 14.

HCl. We notice that the current identification of a square with its side can explain that the wäștum itself is appended, and not a "1" spanned by the wäștum with itself. At the same time we observe that the entity which is "appended" must be the concrete geometric piece of surface, not a number measuring its magnitude: Such a number would, even to the Babylonians, have to be found via one of the "multiplicative" processes "making span" or "raising", as are all "surfaces". Due to the configuration, however, there is no need to "make the wäștum span", i.e. to make it form a rectangle (in fact a square): The square is already spanned by the corner of the cross -- there is no need to prescribe its construction.
102. The specification "to two" shows that the original sense of *ešbum* has been absorbed into the generalization "to repeat $n$ times". Genuine doubling has been left behind.

103. See Rosen 1031:13-15. It cannot be decided on the basis of the al-Khwārizmī’s *Algebra* and the present Old Babylonian text alone whether the recurrence of two Old Babylonian methods in al-Khwārizmī’s *Algebra* is due to coincidence or to continuous tradition. As I shall show in part, however, another algebraic text roughly contemporary with al-Khwārizmī’s shows continuity with the Old Babylonian tradition down to the choice of grammatical forms, while displaying the same interest as the present problem in the four sides of a square and a rectangle. This leaves little doubt that al-Khwārizmī too was inspired by the same old tradition.

104. Diophantus, *Arithmetica* VI, vi. Heron, *Geometria* 21, 9-10. The Diophantine and Heronian parallels have been pointed out by Kurt Vogel (1936:714; 1959:49).

105. This simplification of the geometrical procedure is not in general accompanied by calculatory simplification: the multiplication $\beta \cdot \alpha^{-1}$ is dispensed with, it is true; but the final inverse scaling would be dispensed with in the "Medieval" reduction. (Only cases where $\alpha$ is an irregular which do not divide $\beta$ and $\gamma$ would be harder -- indeed impossible -- to deal with "Medievally"). Of course, such arguments of conceptual simplicity should be used with care. We cannot conclude in that way that Diophantus made use of geometric representations. By his syncopated rhetoric he could keep track of problems much more complicated than the present one. But the Babylonian texts were made neither by nor for mathematicians of Diophantine stature; they were school texts, made for scribe students, comparable in giftedness and interest to the students of Medieval merchant ("abacus") schools, we may guess. If the latter were unable to use the Diophantine method in a rhetorical representation, there is no reason to believe that Babylonian students were any better off.

106. The method is closely related to the method of a "single false position", which was also used by the Babylonians as a purely arithmetical technique -- cf. Kurt Vogel 1960.

107. The same pattern of thought is made explicit e.g. in VAT 7532, rev. 6f.
108. Str. 367 (HKT I, 259f) may be quoted as an example.

109. Expressed in terms of the arithmetico-symbolic representa-
tion aligned with the translation, the former interpretation
makes the variable $x$ the side of the small square -- one seventh
of the side of the first original square. According to the latter
interpretation, $x$ is the ratio between the sides of the original
and the auxilliary squares.

110. Seggs 1960:139.

111. No 2 of the same tablet is a strict parallel -- translated
into symbolic algebra, the condition $x^2 = 9 \cdot (x - y)^2$ is replaced
by $y^2 = 4 \cdot (x - y)^2$. The parallelism makes all restitutions of
damaged passages certain.

I follow the improved readings given by Thureau-Dangin (1936:58,
repeated in TMB).

112. In AO 0862 No 1 (translated below, section VIII.2), even the
inhomogeneous function $xy - x - y$ is a "surface"; so, the meaning
of the term cannot be that of "product". Linear expressions, on
the other hand, are never called surfaces; so, a generalized sense
of "function" or "combined expression" is equally excluded. The
sense "polynomial of the second degree" would of course be adequate,
but much too abstract to be expected in a Babylonian context.

113. Indeed, with one exception, only "surfaces" are "built" in
Old Babylonian algebraic texts (VAT 8390 and AO 0862, in HKT I;
YBC 4608 in HCl; TMS XVII). The exception concerns BM 52801,
the deviations of which from normal usage were already mentioned
above (note 95) -- cf. also below, section X.1.

114. In AO 0862 (see section VIII.2), the calculation is at times
made explicit as a separate process after the construction.

115. The case of BM 15285 No 10 (see above, section V.7) is
different. The whole tablet deals with areas of indubitably geo-
metrical figures; no scaling and no cut-and-paste procedures
appear to be involved.

116. So in VAT 8542 (HKT I, 341, cf. Gondz 1948:36f, or Vogel
1959:72), an auxilliary rectangle is attached to the triangle
spoken of in the enunciation. In this text, by the way, even the
verbal explanation which states the problem is left without the support of a sketch of the situation. Indeed, the problem as stated is clear and unambiguous and requires no sketch. The far less clear exposition of the procedure (less clear at least to modern interpreters) has not given rise to any explanatory drawing.

117. True enough, the association of Archimedes with drawings in the sand are probably due to an Ancient misunderstanding (see Dijksterhuis 1956:30-32). Still, this very misunderstanding shows that geometrical drawings were at times made in the sand. The same is clear from an anecdote told by Vitruvius (De architecture VI, i, the story of the shipwrecked philosopher Aristippus finding geometric figures in the sand of the Rhodian shore).

118. In the Old Babylonian school excavated in Tell ed-Dâr, the exercise tablets of the higher teaching levels contain the instructor's model and the student's attempt to imitate in parallel. The tablets belonging to the elementary level (stylus exercises, "Silbensalphabet A", "Syllabar a") contain no instructor's model, and Tamret (1982:49) proposes that the models have instead been drawn "dans le sable de la cour".

119. Even though Proclus is not very reliable as a source for the early period in Greek mathematics, his statement could be mentioned that arithmetic was first developed by the Phoenicians (In Prima Euclidis ..., Commentarii 653-5).

120. A plan of the fields belonging to the district Šulqi-siš- kalâma, from the tablet HIO 1107, published, redrawn and discussed by Thureau-Dangin (1897).

121. So, the repeated claims of Solomon Gandz (e.g. 1939:415ff), Thureau-Dangin (e.g. TMB, xvii) and Bruins (e.g. THS, 4) that the Babylonians possessed no concept corresponding to our concept of (quantifiable) angles is not contradicted by our field plan. In all probability, the claims are correct for the Old Babylonian period. So, a theoretical concept of the right angle must also be considered absent. But clearly, a practical concept of the right angle, as the correct angle relevant for area measurement, must have existed according to the field plan (and according to much other evidence, including architectural structures and the
expression "the four winds", i.e. four cardinal points). Somewhat pointedly, a Babylonian "right" angle can be claimed to be the opposite of a "wrong" angle.

122. MCT, 44f and Plate 26.

123. I intend to deal with this question in the final version of my "Dynamics, the Babylonians, and Theaetetus 147c7-148d7" (work in progress; the above-mentioned problem is only dealt with in an addendum to the preprint-version circulated in March 1985 (1985b). It should be observed that the multi-digit numbers occurring in many Old Babylonian algebra problems make them unsuited for precise representation through pebble patterns.

124. The verb translated "to collect" in my translation is mešakum, "Ertragungsteil, abgabe einheben". MKT reads mešānum, "einsammeln"; I follow Thureau-Dangin's correction (1936:50), which shows the perspective to be not that of the peasant or the overseer-scribe but that of the landlord or his accountant. Neither this nor Thureau-Dangin's other corrections interferes with the mathematical structure of the text (cf. also MKT III, 58).

125. A concrete interpretation of the procedure was (as far as I know) first proposed by van der Waerden (1961:67).

126. Then the difference in rent would have been calculated e.g. under the two different suppositions that $S_1 = S_2$ and $S_1 = 0$, and the real values of $S_1$ and $S_2$ would have been derived by "inverse linear interpolation". Cf. Tropfke/Vogel 1980:371f.

The difference between the procedure of the present problem and that of a "double false position" was already pointed out by Kurt Vogel (1960:90ff).

127. In those (rather few) cases which go "What shall I pose to $r$ which gives me $x$? Pose $z$, $x$ it gives you", the "raising" of $z$ to $r$ must then be considered as implied by the "posing" as an automatic consequence -- cf. section IV.6.

128. An explanation of the procedure which as far as I know has been overlooked by all previous investigators of the text.

129. See Powell 1972:185 and passim.

130. See Peet 1923:26f.
131. It can be observed that the length of the bar corresponding to the width \( \text{waqum} = 1 \text{ nin dan} \) equals the largest Babylonian length measure, the \( \text{danna} \) (\( \approx 10.8 \text{ km} \)) -- as it was pointed out (independently of the present analysis) by Marvin Powell at the Third Workshop on Concept Development in Mesopotamian Mathematics, Berlin, December 1985.

132. Truly, the mathematical commentary in TMS claims that they do, and even tries to make them do it, though with considerable violence to the texts.

133. According to TMS, only the width is known. Had this been the case, the operations of line A.3 would have proceeded conversely: from a width of 20 to its fourth (5), whence from 45 to 50. In part B, similar disagreements between the text and Bruins’s assumption that the length be unknown can be pointed out.

134. The schematization was proposed to me by Peter Damerow at the first Workshop on Concept Development in Mesopotamian Mathematics, Berlin 1983, where I first presented my interpretation of the text.

135. On other occasions we are of course forced to acknowledge some sumerograms as logograms for proper Akkadian, \( \text{viz.} \) when they are provided with Akkadian phonetico-grammatic complements. Cf. note (a) to BM 13901, No 23 (section V.4).


137. Absent, that is, in explicit formulation. Indications exist, indeed, that the problem BM 85196 rev. II.7-21 was solved as a problem in one variable and not in two, as proposed by Kurt Vogel (1936:710). See my 1985:59f.

138. So YBC 6504 No 2 (MKT III,22, Interpretation in my 1985:44f) and BM 13901 No 19 (MKT III, 4). In both cases, the linear dimensions of the figure are half of those of the present problem.

139. Such a discrepancy is found e.g. in BM 85194, Rev.II,7-21.

140. Similarly, we remember, the "raising" was sometimes left implicit in the "posing" of one number to another (above, section IV.4).
141. It may be significant that two of the three ellipses occur after the "breaking" of a "half-part", which already may imply the construction process; similarly, indeed, in II.21, the half-parts are not "made span" but instead "inscribed until twice". The third ellipse, finally, is found when the area of the square in Figure 17A is found: if this configuration is well-established beforehand, there is no need to construct it anew (cf. the concluding discussion in section V.B).

142. In one case, viz. YBC 4675, obv. 14, ḫaršukum is also used to designate a subtraction from a surface (HCT, 45). That text, however, avoids nasāšum altogether.

143. Truly, Bruins claims in the commentary in TMS (p. 67, and announced already pp. xi and 2) that the two parts deal with the same equation, and that part A expounds the master's own method and part B the alternative used by the Akkadians. For a number of reasons this is an impossible idea:

1) If the equation \( xy+x = 40' \) is to be equivalent with \( xy+2y = 1 \), one must presuppose \( y = 20' \). On the faith of line 6 Bruins claims (rightly, I suppose) that this value will have been given before (cf. my restituted line 1), from which he concludes that the text deals with a normal, complete set of two equations. Line 2, however, presupposes implicitly that the length is equally known (10' the surface), while the value is stated explicitly in line 5 (still without being calculated).

2) If the first half of line 10 were to be the result of a transformation belonging with the "Akkadian method", it could not precede the announcement of that method in the second half of the line.

3) In any case, the first half of line 10 is clearly in the style of statements; transformed equations are never restated in a similar form -- cf. the fact that part A does not end up with a new statement "length and width accumulated, 1'50', 40' the surface"; cf. also the contrast with the formulation in lines 25f.

4) Finally, Bruins overlooks the identical statement in part C, as well as the fact that the procedure taught in part B is precisely the one used in part C.

It may be observed that the presumed "Susian" method is used in the Babylonian ("Akkadian") AO 8362 no 1-2, although no 2 would have been greatly simplified had the "Akkadian method" been used.
146. In this connection, the global character of Old Babylonian scribe school mathematics is worth reflecting upon. Greek mathematics, that other prototype of Ancient non-utilitarian mathematics, can be claimed to be essentially determined by its central problems (squaring the circle, doubling the cube, properties of conics, classification of irrationals, etc.). The great methodological innovations of Greek mathematics were made in order to solve (in a philosophically satisfactory manner!) these great problems. Old Babylonian scribe school mathematics was (in as far as we concentrate upon its non-utilitarian aspect) determined by the methods at hand, and problems were chosen that would permit a brilliant display of the methods known to "the learned scribe", which makes scribe school mathematics a perfect parallel to other aspects of Old Babylonian scribal culture as presented e.g. in the "examination texts". See my 1965c:10-16 and passim, which discusses the difference/less coarsely than necessary in the space of a foot-note, and connects the different attitudes to their institutional context.

147. The meticulous repetition of all steps appears to exclude a simple reference back to the known entities from section B.

146. The argument can be imagined in the style of "false assumptions": if the upper left rectangle in Fig. 168 is to represent 27 "true" lengths, the length of the upper right rectangle is J instead of 27. Similarly, if the upper left width represents 27 "true" widths, its extension will have to be 27 instead of 27. The sum of length and width of the total figure will then be 3 + 27 = 27.30", cf. line 34. Furthermore, the total scaling factor for the area will be 27.3 = 3, and the area of the assumed surface will hence be 1.3 = 2.3.2 (lines 36-39).

The last part of the interpretation seems to be confirmed by VAT 8520, No 17.4.M.4, an 1qum-igibum problem (translatable into xy = 1, x-y(x+y) = 30'). is solved in a similar way (extensions apart). The linear equation is transformed, it appears, into 7x-6y = 8.30', and a scaling factor of 7:6 = 42 is applied to "the surface". As the numbers 7 and 6 are to be retained by head, the transformation can be assumed to be performed mentally, not by means of any material representation beyond the changed conceptualization of the basic rectangle.
147. This supplementary role is nothing specific for the mathematical texts. Similar claims could be made for most branches of Babylonian literature.


149. It should perhaps be emphasized once more that these remarks, as the whole of my investigation, regards the "algebraic" texts. They have no implications for those texts which are directly concerned with the properties of numbers, e.g. concerning inversion or continued multiplication; they, of course, cannot be denied the label "arithmetical".

150. Inclusion of certain further texts would have forced us to modify this statement as well as the automatic identification of "surface"-problems and problems of the second degree. So, the "surface" problem Str. 367 (HK I 1, 259f) is in reality of the first degree, but makes use of certain naive-geometric techniques all the same; other exceptions of various sorts could be mentioned. (Already the first-degree "meadow" problems of VAT 8389 and 8391 could indeed be claimed to be exceptions; all of them are of the first degree -- but formally they are of course concerned with surfaces, and part of the reasoning is made through imagined partition of a geometrical surface).

Problems "representing" prices, igûm-igûm pairs etc. by dimensions of surfaces are not to be understood as exceptions but as "surface"-problems -- cf. the use of the term "surface" in YBC 6967 (above, section V.1).

151. A very beautiful example is VAT 8512(MKT I, 341f) -- see Gendz's deciphering of the procedure (1948:36), or the more detailed analysis of the text in my 1985:105.15ff.

152. This would hardly bother the Babylonians, who appear to treat a rectangle of length 45 nindan and height 45 cubit as they treat an ordinary square -- see my 1985:53-63.

153. Since our texts are school-texts and not practitioners' notebooks this may seem their only possible aim. The occurrence of problems of the third degree (for which the Babylonians knew no general solution, and which are therefore treated by non-generalizable tricks) show that another aim was possible and in fact also present at least occasionally: that of demonstrating the (mock) ability of the teacher. Cf. also above, note 144.
154. Seen in a long-run perspective this is of course also true of modern mathematical terminology. New theoretical developments give rise to new applications of old terms -- just think of a creature like the "infinite-dimensional vector space", in which at most "infinity" can still claim a classical value. Since the time when mathematical terms were given precise definitions, however, every extension by analogy and metaphor constitutes a clear and definite break. This was apparently different in Babylonian mathematics, which saw no absolute conceptual border-line between standard-situation and analogous extension.

155. This is the most probable implication of the distinction between "length" and "true length" in TMS XVI (section VII.3). The "squared line" spoken of in the statement and that inherent in the procedure can also be seen to be kept apart in BM 13901 No 14 through the multiplication by \( \sqrt{2} \) in rev.1.9 (section VIII.1). Finally, TMS XIX appears to designate a "representing length" as the "counterpart" of the "real length" (cf. below, note 176).

Abstract distinction between a mentally conceived "real entity" and an equally mental "representing entity" may be too abstract to be expected in a Babylonian context. A reasonable guess would be that the traces of an explicitly distinguished representation are also traces of a concrete material representation.

156. There is no reason to be overly astonished (or scandalized on behalf of the poor scribe school students) on this account. Apart from a modest (not to say infinitesimal) minority of the school children who have been taught second-degree algebra during the latest 3½ millennia, their situation has been exactly the same, when not worse. Unless you make interpolation in trigonometrical or similar tables, physics at least at the level of Galilean ballistics, or something similar, second-degree algebra can only be used to train second-degree algebra.


158. Chapter 1, quoted from Reich & Gericke 1973:37. Vieta cites Theon's definition of analysis. The cold metaphor is found in the dedicatory letter (ibid. p. 34).

159. We observe that even the argument by false position is a primitive sort of analysis albeit arithmetical. Take e.g. the problem that a "heap" and its fourth is 15. For lack of an \( x \) permitting us to rewrite the 15 as \( \frac{1}{4} x \) one takes the number to be known, viz. 60, etc.

161. Precisely this question is raised for Babylonian mathematical thought by Mahoney (1971:370).

162. That even large parts of mythology were founded on social practice has been argued by Thorkild Jacobsen (1976; and already in Frankfort, 1946:125-219). A proverb like "Workmen without a foreman are waters without a canal inspector" demonstrates clearly that Babylonian scribes were as able to see their fellow beings under the aspect of objects as their myths were to see nature as a fellow being (Frankfort et al 1946:203).


164. Frankfort et al 1946:5.

165. Ibid.

166. See Renger 1976:229 and passim.


169. The opposition between day and night can thus be used as an analogy or "model" for the two moieties of a tribe; clans labeled after animals are part of common lore -- not signifying, however, that the clan members assume descent from the real animal in question, but affinity in some higher sense (cf. Ibid. pp. 142f and 149).

170. Ibid. p. 20

171. The definitions, axioms and postulates of the Elements are precisely such a set of abstracted principles, and the build-up of the whole work constitutes a conscious attempt to build the complete argument on these. Truly, the abstracted system is not complete, as it is well known, and at times "naive" knowledge is made use of implicitly; and conversely it is obvious that Old Babylonian "naive" geometry is full of implicit abstraction: assumptions on the calculability of areas as products, knowledge of arithmetical rules, etc. Neither observation affects the fact that we have to do with fundamentally different projects.
173. The system is clearly visible in the symbolic transcriptions of three sections of VAT 7537 in MKT I, 474f.

174. For the motivations of Old Babylonian non-utilitarian mathematical activities, cf. above, note 144. The changes after the end of the Old Babylonian era are discussed in my 1980:28f.

175. From the mathematical structure alone, Bruins' interpretation, viz. a triangle cut by a transversal, cannot be excluded. But the expression "upper length" in rev. 17 speaks definitely against it, as does the absence of partial areas from the statement.

176. "Normally", but not exclusively, it is true. In TMS XIX, a number 1 is posed (in a "single false position") for the (real) "length" of the problem, and next also for its "counterpart" (TMS, 101, as corrected in von Soden 1964:49), which in the following turns up to be its "basic representative". In TMS IX, 40 (above, section VIII.3) as well as TMS XII, 10 (TMS, 79, as corrected in von Soden 1964:49), and as rev. 5 of the present text, the "original" and the "counterpart" form the usual geometric configuration, but already at the point where they are "made span" a supplementary square, not when the side of the supplemented square is found.

An occurrence in IM 55357, (Baqr 1959) is still more deviant but need not occupy us here, as it has to do with a triangle.

177. The same expression is found in the contemporary and equally northern tablet Haddad 104 (al-Rawi & Rosaf 1985) and in the late Old Babylonian or perhaps even early Kassite AO 17264 (MKT I, 126). Db.-146 (Baqr 1962), which is also contemporary with the present text, regards the "equilateral" as something which is to be "taken" -- presumably also as a numerical result.

178. Another case of re-Sumerianization is that of tab. In Old Babylonian mathematics, it was used as a logogram for sēpqum, "to double"; in the present tablet (e.g. obv. I.2) it is used for tepām, "to add". In view of the genuine meaning of Old Babylonian "doubling" (concrete repetition), both uses are in agreement with the general meaning of the Sumerian term; in their technical use, however, the two functions of the ideogram cannot be connected as in any way, which excludes any continuous existence as a mathematical term.
179. Of course in symbolic transcription (MKT 111, 21). The important thing is that the entity \((1+w+d)^3\) cannot be avoided in the interpretation of the procedure.

180. A0 6484, the other Seleucid tablet containing second-degree problems, was written by Anu-abu-ma-tār, an early 2nd-century scribe from Uruk, known as possessor and writer of astronomical (and other) tablets. See the colophon in MKT I, 99, and Hunger 1968:40 (no 92) and passim.

If the algebraic tradition was really transmitted since the Old Babylonian period in an environment of "higher artisans", as suggested above, the circle of the Uruk astronomer-priests may be the setting where its reSUMERIANIZATION took place.

181. In Old Babylonian mensuration, the area of an irregular quadrangle had been found by the "surveyor's formula", as the product of "average length" with "average width" -- see e.g. YBC 4675, in MCT, 44f). In the Seleucid tablet VAT 7848 (MCT, 141) the height of a trapezoid is calculated by means of the Pythagorean theorem, and everything goes exactly as in Hero's Geometrica 16,17.

182. Rev. 10-27 (4 problems in total). In MKT I, 98f.

183. The subtraction of "surfaces" carries a libbi, "inside"; but the subtractive term itself is la 1, "diminish", and the addition is expressed simply by y, "and", and tab, "add". Multiplication is seen as "going steps".

184. Translated by Kurt Vogel (1968 -- the relevant problems are found pp. 90-103).

185. One source is a Greek papyrus from the 2nd century A.D. (Rudhardt 1970, cf. Sesiano 1986. Another is a Latin Liber podium (latest edition in Bubnov 1899:510-516) dating perhaps to the 4th century A.D. and based apparently on Alexandrinian sources. One of its problems (ibid. p. 511f) deals with a right triangle, for which the hypotenuse and the area are known. The solution is of "Seleucid" type, making use of total sum and total difference.

186. I shall only refer to Szabó 1969; Mahoney 1971, and Unguru and Roe 1981.


188. See my 1983b.
189. The first Arabic passage is grammatically impossible as it stands. Rashed (1984:113, 27) prefers a minimal correction, which makes the term an epithet to a number. Sesiano (1982:99 note 48) makes a more radical emendation, in order to obtain agreement with a backward reference in the next passage and with his own interpretation of the term. The first but not the second of these considerations seems compelling to me, which makes me accept that part of Sesiano’s correction which makes IV.17 a parallel to IV.19 (whence also to V.7).

190. Ver Eecke 1926:36 note 6. There is no reason to go further into the details of his explanation.


192. See my 1986.


195. See Luckey 1941.

196. One may wonder that so many linguistic observations can be made on a Medieval Latin translation. The reason is that Gherardo’s translation appears to be extremely literal, reflecting even some peculiarities in the original usage which could easily have been straightened without loss of mathematical substance.


198. See Krasnova 1966:115ff. This Russian translation is the only printed version of the work, although selections and paraphrases from incomplete manuscripts have been published by Woepcke (1855) and Suter (1922:94-109). Though not algebraic the whole treatise is highly interesting as an eclectic merger between a Near-Eastern naive-geometric tradition and Greek apodictic geometry.

And Qal'WarB's treatise is a main source for the establishment of a connection between the cut-and-paste technique and the later theory of partition if figures.
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<th>Assyrian</th>
<th>meanings</th>
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<td>kamarrum / gar-gar / UL.GAR</td>
<td></td>
</tr>
<tr>
<td>accumulated</td>
<td>nakarrum</td>
<td></td>
</tr>
<tr>
<td>accumulation</td>
<td>kunarrum / gar-gar / UL.GAR</td>
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</tr>
<tr>
<td>add</td>
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<tr>
<td>Akkad</td>
<td>akktānum</td>
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<tr>
<td>and</td>
<td>u</td>
<td></td>
</tr>
<tr>
<td>append, to</td>
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<td></td>
</tr>
<tr>
<td>appended</td>
<td>wguubbūnum</td>
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<tr>
<td>as</td>
<td>inūna</td>
<td></td>
</tr>
<tr>
<td>as much as</td>
<td>kīna</td>
<td></td>
</tr>
<tr>
<td>ask, to</td>
<td>ḫālum</td>
<td></td>
</tr>
<tr>
<td>become: small(er), to</td>
<td>majōm</td>
<td></td>
</tr>
<tr>
<td>break, to</td>
<td>heqānum / qaz</td>
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</tr>
<tr>
<td>break off, to</td>
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<td>wabānum</td>
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<td>build, to</td>
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<td></td>
</tr>
<tr>
<td>by</td>
<td>inā / -ta</td>
<td></td>
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<tr>
<td>change</td>
<td>taktītum</td>
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<td>collect (taxes, rent), to</td>
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<tr>
<td>confront, to / -ed</td>
<td>UL.UL</td>
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<td>panānum</td>
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<td>cubic equilateral</td>
<td>ba-si₄/si</td>
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<tr>
<td>cubit</td>
<td>amānum / kuš</td>
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<td>kaštum</td>
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<tr>
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<td>nakānum / kud</td>
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</tr>
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<td>cut off, to</td>
<td>barānum</td>
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<td>detach, to</td>
<td>patārum / du₄</td>
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<tr>
<td>double, to</td>
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<td>each</td>
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<td>equilateral</td>
<td>i₄-ba-si₄/si / barūnum / ba-si₄/si₄</td>
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<td>lūl</td>
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<tr>
<td>first ... second</td>
<td>šarrūm</td>
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<tr>
<td>(1st ... 2nd)</td>
<td>l(k a₄m) ... 2(k a₄m)</td>
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<tr>
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<tr>
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<tr>
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<tr>
<td>Front</td>
<td>iāqūm / gīn</td>
<td></td>
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<td>Gin</td>
<td>nādēnum / sum</td>
<td></td>
</tr>
<tr>
<td>Give, to</td>
<td>alēkum / rā</td>
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</tr>
<tr>
<td>Go, to</td>
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<td></td>
</tr>
<tr>
<td>Go away, to</td>
<td>waṭīrum / dirīg / si</td>
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<tr>
<td>Go beyond, to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grain</td>
<td>āšūm / āe</td>
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<td>Gur</td>
<td>gūs</td>
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<td>Half</td>
<td>mišūm / u-ru-ri-a</td>
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<tr>
<td>Half-part</td>
<td>bāntum / BA.A</td>
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<td>Having-been-made</td>
<td>nēšānum</td>
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<tr>
<td>Head</td>
<td>rēsām / saq</td>
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<tr>
<td>Head retain, may your</td>
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<td>Here</td>
<td>rēsāka likil</td>
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<td>If</td>
<td>annākt'ām</td>
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<tr>
<td>Iqīn</td>
<td>likū / igi</td>
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<tr>
<td>Inscribe, to</td>
<td>lapītum</td>
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<tr>
<td>Inside</td>
<td>libīnum (in the inside of: libba; inside of: libbi)</td>
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<tr>
<td>Integrity</td>
<td>šalānum</td>
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<tr>
<td>Itself</td>
<td>rāšnīnā</td>
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<td>Know, to</td>
<td>sūdām / zu</td>
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<td>Lay down, to</td>
<td>nādām</td>
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<td>Leave, to</td>
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<td>nām</td>
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<td>Lower</td>
<td>kāpūm / kī(-ta)</td>
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<td>Make, to / making</td>
<td>epērum / kl</td>
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<tr>
<td>Make cubic equilateral</td>
<td>(-e) ba-sī/sī</td>
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<tr>
<td>Make equilateral</td>
<td>(-e) in-sī/sī</td>
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<td>Make follow (additive-ly), to</td>
<td>ruddām (D-stem of radūm)</td>
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<td>Make span, to</td>
<td>šutūtum / l-kū (-kū)</td>
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<tr>
<td>Make surround, to</td>
<td>nīghīn</td>
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<tr>
<td>English</td>
<td>Akkadian</td>
<td>Notes</td>
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<tr>
<td>---------</td>
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<tr>
<td>meadow</td>
<td>tawīrum / ģarīm / (A-ENCUR?)</td>
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<tr>
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<td>manūm</td>
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</tr>
<tr>
<td>name</td>
<td>ėnumum (Seleucid MU)</td>
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<td>nīndan</td>
<td>nīndān (=GAR)</td>
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<td>la / nu</td>
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<tr>
<td>no: (negating a proposition)</td>
<td>ul(a) / nu</td>
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<tr>
<td>now</td>
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<td>one</td>
<td>ītēnum</td>
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<tr>
<td>one ... the other</td>
<td>ītēn ... ītēn</td>
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<tr>
<td>oppose, to</td>
<td>lūtu</td>
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<tr>
<td>out from</td>
<td>ītu</td>
<td></td>
</tr>
<tr>
<td>over</td>
<td>āli / u-ya</td>
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</tr>
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<td>over-going</td>
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<tr>
<td>part, n'th</td>
<td>līginigal(-bli)</td>
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<td>pose, to</td>
<td>ṣākānum / gar</td>
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<td>raise, to</td>
<td>nēšūm / 11</td>
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<tr>
<td>raise against itself, to</td>
<td>ṣātānum</td>
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<tr>
<td>reed</td>
<td>qēnum / gi</td>
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<td>remain, to</td>
<td>(Seleucid; rēhum)</td>
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<td>retain, to</td>
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<td>ser</td>
<td>s-ar</td>
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<tr>
<td>say, to</td>
<td>qābām / ẖuq,</td>
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<tr>
<td>saying</td>
<td>ḫuq, / TUK</td>
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<tr>
<td>second / 2nd</td>
<td>šdānum / 2(śanum)</td>
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<tr>
<td>see, to</td>
<td>šāpirum, cf. &quot;you see&quot;</td>
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<td>seventh</td>
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<td>sīla</td>
<td>qa / sīla</td>
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<td>since</td>
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<td>sixty</td>
<td>šuḏūn</td>
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<td>small</td>
<td>see “be(come) small(er)”</td>
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<tr>
<td>so</td>
<td>kīm</td>
<td></td>
</tr>
<tr>
<td>so much as</td>
<td>nēla / a-na</td>
<td></td>
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<tr>
<td>span</td>
<td>see “make span”</td>
<td></td>
</tr>
<tr>
<td>squared line</td>
<td>mithartum / LAGAB</td>
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</tr>
<tr>
<td>steps of</td>
<td>a-rá</td>
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</tr>
<tr>
<td>surface</td>
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<td>surrounding</td>
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<td>takiltum</td>
<td>takiltum</td>
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<tr>
<td>tear out, to</td>
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<td>third (part)</td>
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<tr>
<td>turn back, to</td>
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<tr>
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<td>two</td>
<td>āsina</td>
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<td>two-third</td>
<td>āsinīptum</td>
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<tr>
<td>upper</td>
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<td>wēštam</td>
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<td>wēšom</td>
<td>wēšum</td>
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<td>what</td>
<td>minūm / en-nam</td>
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<td>ā āe</td>
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<tr>
<td>width</td>
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</tr>
<tr>
<td>you see</td>
<td>tamār / īši-di-ba/du</td>
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</tbody>
</table>

* The table is intended to be comprehensive with regard to the texts translated above. Only pronouns and pronominal suffixes are left out intentionally. The table includes a number of terms which were not represented in the above translations, but which would be useful for other texts belonging to the genre. For this open-ended enterprise, no completeness was of course aimed at. It should be emphasized once more that this is a table of standard translations, i.e. a key to the translated texts. It is not meant to be a dictionary, and no listing of meanings.
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<th>Akkadian</th>
<th>English</th>
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<tr>
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<td>(tarik-</td>
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<td>(tum?)</td>
<td>tum)</td>
<td></td>
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<td>Akkadian</td>
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<tr>
<td>akkūnu</td>
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</tr>
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<td>akkūrum</td>
<td>to see</td>
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<tr>
<td>akkūram</td>
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<td>ašir'án</td>
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<td></td>
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<tr>
<td>ašá (aša-l)</td>
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<tr>
<td>ašum</td>
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<td>ašum</td>
<td>since</td>
<td></td>
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<tr>
<td>ba (aši) (bāntum)</td>
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<td></td>
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<tr>
<td>ba (aši) ta (bāntum)</td>
<td>cubic (cubic) equi-</td>
<td></td>
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<tr>
<td>bāntum (ba, aši)</td>
<td>half-part</td>
<td></td>
</tr>
<tr>
<td>banûm</td>
<td>to build</td>
<td></td>
</tr>
<tr>
<td>banûm</td>
<td>half-part</td>
<td></td>
</tr>
<tr>
<td>bù esin (bùr-rum)</td>
<td>bùr</td>
<td></td>
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<tr>
<td>bù rum (bù-rum)</td>
<td>bùr</td>
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</tr>
<tr>
<td>daš (daš-šum)</td>
<td>to append</td>
<td></td>
</tr>
<tr>
<td>i-šu (i-šu i-šum)</td>
<td>(over ...) go beyond</td>
<td></td>
</tr>
<tr>
<td>dāš (dāš-šum)</td>
<td>to detach</td>
<td></td>
</tr>
<tr>
<td>dāš (dāš-šum)</td>
<td>to say/saying</td>
<td></td>
</tr>
<tr>
<td>ede (a u)</td>
<td>to know</td>
<td></td>
</tr>
<tr>
<td>edâru</td>
<td>over-going</td>
<td></td>
</tr>
<tr>
<td>edî (a u-gu)</td>
<td>over</td>
<td></td>
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<tr>
<td>edûn (a-n)</td>
<td>upper</td>
<td></td>
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<tr>
<td>edûn</td>
<td>to come up (as a result)</td>
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<td>what</td>
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<tr>
<td>ṣ̣ep̣̣ānum (aš-a)</td>
<td>to make/making</td>
<td></td>
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<td>TUK, see dug,</td>
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<tr>
<td>zu (-edûm)</td>
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"Cf. note 3) to Table 2. Only logographic equivalents testified in mathematical texts are listed. In the translations of the texts, each term is written in the same typography as the transliteration of the term it translates."
Abbreviations

AHw W. von Soden 1965, Akkadisches Handwörterbuch.
GAG W. von Soden 1952, Grundriss des akkadischen Grammatik
HAMw W. Gesenius 1916, Hebräisches und Aramäisches Handwörterbuch.
NCT O. Neugebauer & A. Sachs 1945, Mathematical Cuneiform Texts.
NEA  R. Lebät 1963, Manuel d'épigraphie élamienne.
MKT O. Neugebauer 1935, Mathematische Keilschrifttexte I-III.
RA Revue d'Assyriologie et d'Archéologie Orientale.
SL A. Deimel 1925, Sumerisches Lexicon I-III.
TMB F. Thureau-Dangin 1938, Textes mathématiques babyloniens.
TMS E.M. Bruins & M. Rutten 1961, Textes mathématiques de Suse.
ZA Zeitschrift für Assyriologie und vorderasiatische Archäologie.

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